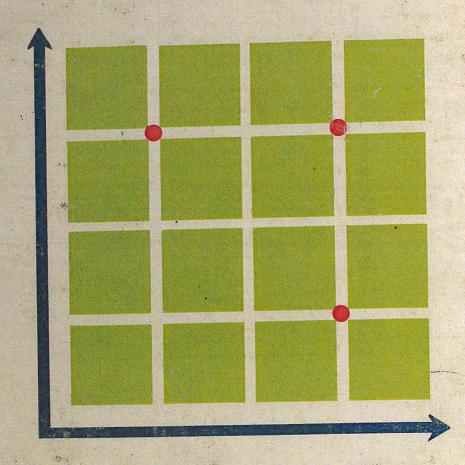
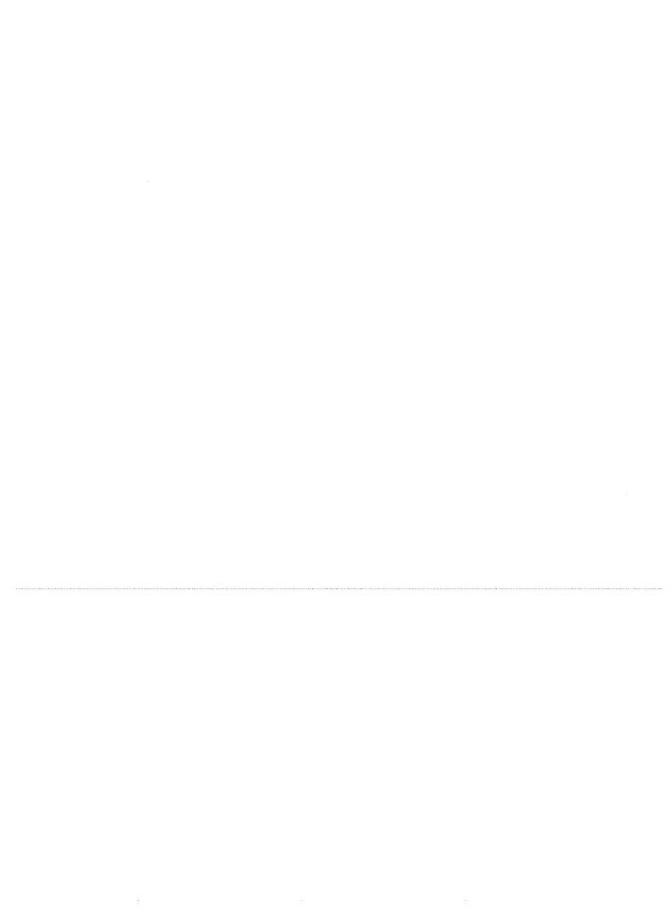
# Modern School Mathematics Structure and Use







## **Modern School Mathematics**

Structure and Use

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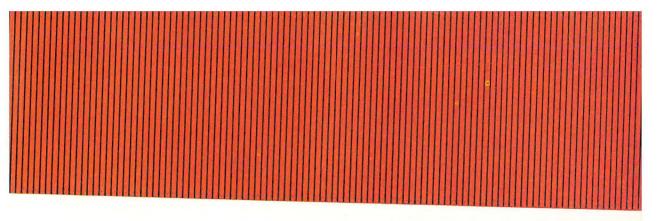
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# Modern School Mathematics Structure and Use

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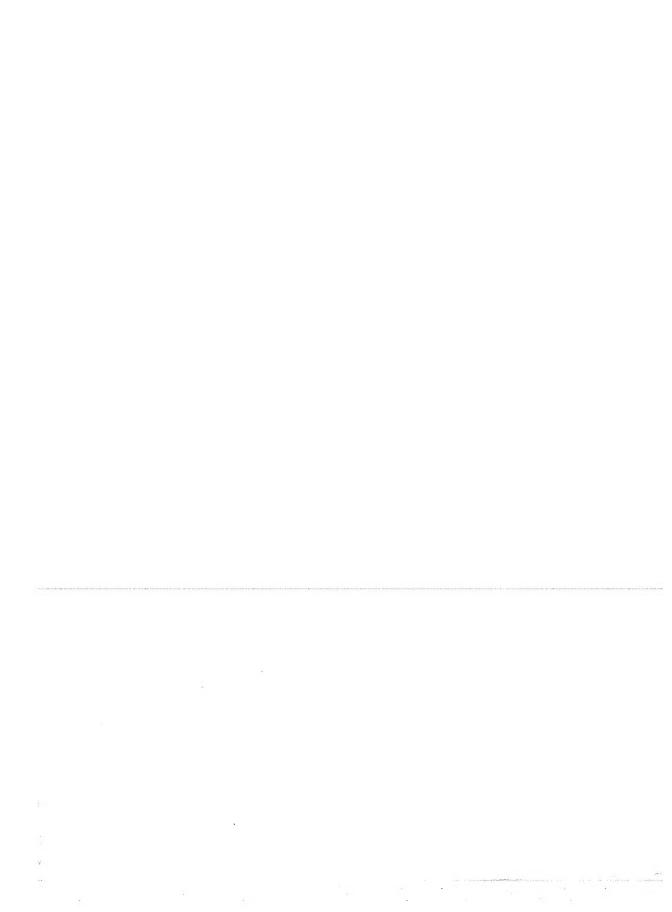
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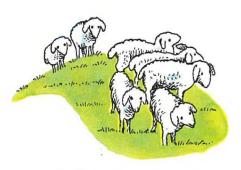
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# CHAPTER \* Sets, Numbers, Numerals

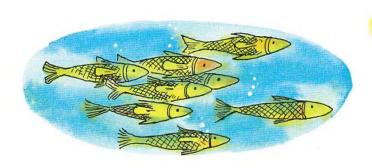
#### Sets



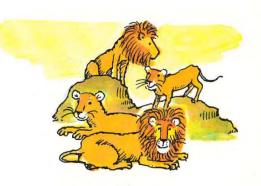
A flock of sheep



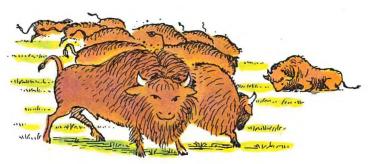
A gaggle of geese



A school of fish

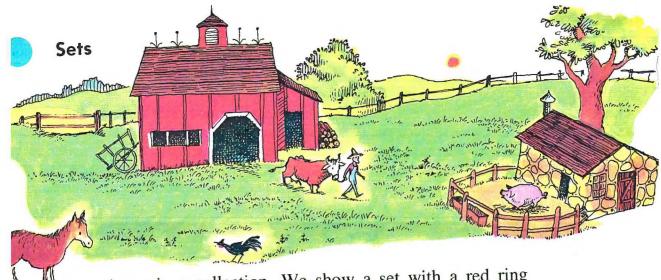


A pride of lions



A herd of buffalo

Each of these is a set. Can you think of other sets?



A set is a collection. We show a set with a red ring like this or with braces like this { }. We can show what is in a set in two ways:

Listing the objects in a set: {a chicken, a cow, a horse, a pig}.

Describing the objects in a set: {Farmer MacDonald's animals}.

#### EXERCISES

Show these sets by **listing** the objects. The first one is done for you.

- 1. {the first five letters in the alphabet}
- 2. {the names of the months of the year that begin with M}
- 4. {the name of the town and the state where you live}

Answer  $\{a, b, c, d, e\}$ 

- 3. {the names of the children in the front row of your class}
- **5.** {the numerals on the face of a clock}

Show these sets by describing the objects in them. The first one is done for you.

- **6.** {spring, winter, fall, summer}
- 7.  $\{a, e, i, o, u\}$
- (Monday, Tuesday, Wednesday)

Answer {the four seasons in the year}

- 8.  $\{u, v, w, x, y, z\}$
- 10. {January, February, March}



#### EXERCISES

Write the numerals 1, 2, 3, 4. Next to each numeral write K, L, M, or N to show the set on the right which is a subset of the set on the left.







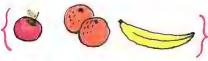


 $\{1, 2, 3, 4, 5\}$ 

M.



4.



N.

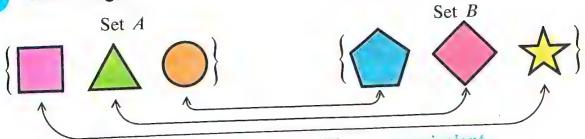


- 5. Draw 7 apples like this one subset of 5 with red, and the remaining subset with blue.
- **6.** Draw 8 shapes like this one  $\diamondsuit$ . Color a subset of 6 red, a subset of 1 blue, a subset of 1 green.

Here is a set of names: {Kathy, Karen, Karl, Mary, Mark, Martha}. Write the subset of names that:

- 7. begin with K
- 8. begin with M
- 9. end with y

#### Matching, one-to-one



These sets match one-to-one. They are equivalent. They have the same number of objects.

The number of objects in a set is called the cardinal number of the set.

#### EXERCISES

1. Write the letters A, B, C on your paper. Next to each letter write R, S, or T to make pairs of equivalent sets.

$$A = \{\text{Bill, Dick, Tom,} \\ \text{Fred, Harry}\}$$

$$B = \{\triangle, \triangle, \triangle\}$$

$$C = \{\text{a, b, c, d, e, f, g}\}$$

$$R = \{\triangle, \triangle, \bigcirc, \bigcirc, \bigcirc\}$$

$$S = \{\text{the days of the week}\}$$

$$T = \{\Box, \Box, \Box\}$$

- 2. If two sets M and N match one-to-one, and there are 8 objects in M, how many objects are there in N?
- 3. Show 3 ways of matching the sets  $\{\Box, \triangle, \bigcirc\}$  and  $\{ \not \leftarrow, \nabla, \spadesuit \}$ .
- 4. Name the cardinal number of each set in the first exercise.

{ } has no objects. It is the empty set. What is the cardinal number of the empty set?

5. {All the elephants in our classroom} is the empty set. Name another example of the empty set.

#### Greater than—less than



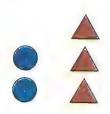
One fork does not have a knife to match.

7 is less than 8

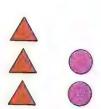


One child does not have a chair to sit on.

6 is greater than 5



2 is less than 3



3 is greater than 2 3 > 2

#### EXERCISES

What is the cardinal number of each of these sets?

**2**. *B* =





4. D={

Using the sets in exercises 1-4, complete these statements. Write < or > to replace the O.

**5.** Set A is smaller than set B.

So 3 () 4.

**6.** Set C is larger than set D.

So 6 ( ) 5.

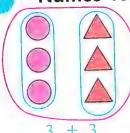
7. Set D is  $\underline{?}$  than set B.

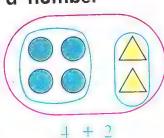
So 5 ( ) 4.

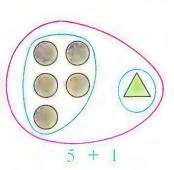
**8.** Set A is  $\underline{\phantom{a}}$ ? than set C.

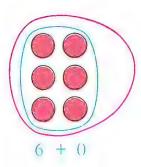
So  $3 \bigcirc 6$ .











0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are digits. Digits are used to write numerals. Numerals are names for numbers. Each number has many names. 3 + 3, 4 + 2, 5 + 1, 6 + 0 and "six" are names for the same number.

#### EXERCISES

Write the word **number** or **numeral** to complete these sentences.

- 1. Bob is thinking about a \_\_?\_.
- 2. He is writing a ?

Write the numerals.

- 3. 5 is a name for a number. Write 4 other names for this number.
- **4.** 9 is a name for a number. Write 4 other names for this number.
- 5. Name the greatest number that can be shown with one digit.

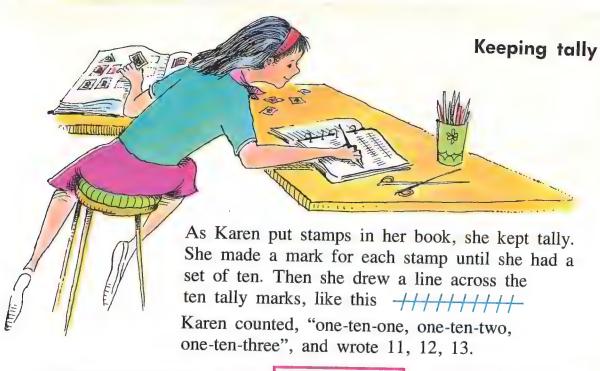
Write the numerals on the right that name the same number as the numeral on the left.

Write a one-digit numeral for each of these numbers.

9. 
$$3 + 4$$

12. 4 + 5

11. 
$$2 + 2 + 3$$



#### EXERCISES

Use Karen's tally system to show:

- 1. 1 ten and 4 more
- 3. 3 tens and 2 more
- 5. 1 ten and 8 more

- 2. 2 tens and 5 more
- 4. 4 tens and 6 more
- 6. 5 tens and 1 more

Write 2-digit numerals for the numbers which Karen called:

- **7**. <del>||||||</del> |||||
- 9. one-ten-seven
- 11. three-ten-seven
- 13. four-ten-seven
- 15. five-ten-seven

- 10. two-ten-three
- 12. one-ten-eight
- 14. four-ten-one
- 16. six-tens

Write the names which Karen would have given these numbers.

- **17**. 14
- **21**. 42
- **25**. 16
- **29.** 71

- **18.** 18
- **22.** 59
- **26.** 28
- **30**. 64

- **19**. 24
- **23**. 80
- **27.** 57
- 31. 9

- **20.** 36
- **24.** 97
- **28**. 8
- **32**. 11

(seven) 7

## Four ways of showing the same number

Tens	Ones	Tens	Ones	Expanded numeral	Short form
111	////	3	4	30 + 4	34

We read 34 as 3 tens and 4 ones, as thirty plus four or as thirty-four. The digit 3 is in the tens' place, the digit 4 is in the ones' place. 30 + 4 is called an expanded numeral. The short form is 34.

#### EXERCISES

A

 $\mathbb{C}$ 

0

X

В

c + n

u + v

Complete the pattern we have begun here. Write A, B, and C at the top of your page and write the numerals as we have done for the first example.

	Tens	Ones	Expanded numeral	Short form
1.	2	3	20 + 3	23
2.	z		r + e	n
3	a	C	e + n	0
4	r	S	u + v	x

|z|

r

|a|

|S|

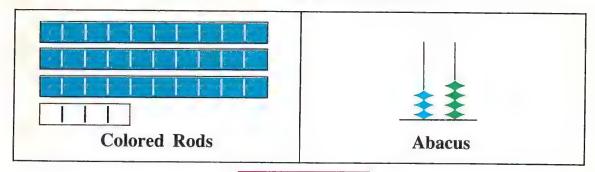
5.

6.

#### More ways of showing numbers

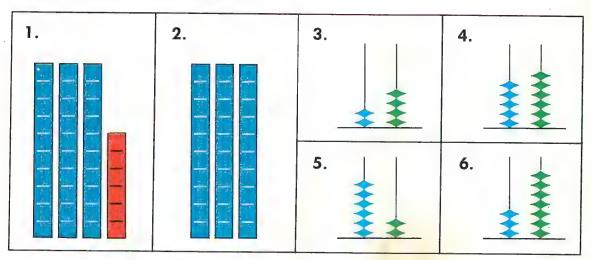


Here are two other ways of showing the number 34.



EXERCISES

Name the numbers shown here.



Write the short form for:

7. thirty-four

8. eleven

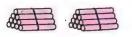
9. twenty-nine

- **10.** eighty-three
- 11. forty-six

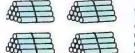
- 12. sixty-eight
- 13. Write expanded numerals for the numbers in exercises 7 12.

Write the short form for each of these numbers.

14.



15. 🍇



16.



17.

Tens Ones 1111 11111

18.

Tens Ones

19. 20 + 7

#### Counting numbers

Barbie was asked to count the children in her classroom. She began, 1, 2, 3, and ended with 28.

The numbers she used are called

#### counting numbers

The counting numbers are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and so on.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

We can show the counting numbers on the number line.

#### EXERCISES

Use the number line to help you to:

- 1. name the counting number that comes just after **a.** 12 **b.** 16 **c.** 20 **d.** 4 **e.** 24

- 2. name the counting number that comes just before

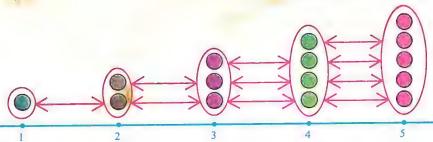
  - a. 14 b. 17 c. 21 d.
- 6 e. 23
- 3. name the counting number that comes between
  - **a.** 5 and 7. **b.** 15 and 17 **c.** 19 and 21

21 22 23 24 25 24 17 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42

Use the number line to help you to:

- 4 name the counting number that comes just after
  - a. 31
- **b**. 29
- c. 40
- d. 25
- **e.** 28
- 5. name the counting number that comes just before
  - **a.** 30
- **b.** 40
- c. 36
- **d**. 25
- e. 22
- 6. name the counting number that comes between
  - **a.** 27 and 29
- **b.** 40 and 42
- **c.** 29 and 31
- 7. name the counting number to complete each set of numbers in order.
- **a.**  $\{40, \ \overline{a}, \ 42\}$  **b.**  $\{3, \ 4, \ \overline{e}, \ 6\}$  **c.**  $\{27, \ 28, \ \overline{c}, \ 30\}$

#### Greater than, less than



The counting numbers on the number line are in order. The greater numbers are to the right.

#### EXERCISES

Name the counting number that is 1 greater than:

- 1. 3
- 2. 7
- **3**. 17
- 4. 8
- **5**. 19
- **6.** 20

- **7**. 0
- 8. 14
- **9**. 23
- **10.** 41
- 11. 99
- **12**. 31

Name the counting number that is 2 greater than:

- **13.** 0
- 14. 12
- **15**. 8
- **16**. 14
- **17.** 30
- **18.** 43

- **19.** 3
- 20. 39
- **21.** 48
- **22.** 61
- **23.** 18
- **24.** 29

Name the counting number that is 1 less than:

- **25.** 10
- **26**. 3
- **27.** 24
- **28.** 17
- **29.** 100
- **30.** 38

- **31.** 31
- **32.** 28
- **33**. 49
- **34.** 86
- **35.** 99
- **36.** 60

Name the counting number that is 2 less than:

- **37.** 43
- **38.** 12
- **39.** 51
- **40**. 17
- 41. 20
- **42**. 14

- **43**. 29
- **44.** 36
- **45**. 81
- **46**. 12
- **47.** 51
- **48**. 42

Name the counting number that:

- **49.** is greater than 6 and less than 8.
- **51.** is less than 9 and greater than 7.
- **53.** is greater than 4 and less than 6.
- **55.** is less than 18 and greater than 16.

- **50.** is greater than 0 and less than 2.
- **52.** is greater than 12 and less than 14.
- **54.** is greater than 22 and less than 24.
- **56.** is less than 32 and greater than 30.







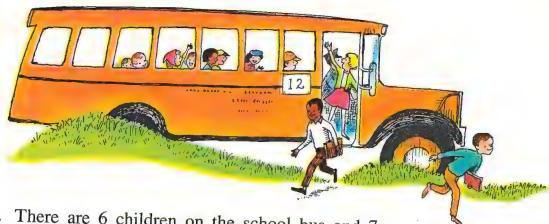
Bobo was first. Sandy was second. Shorty was third.

#### EXERCISES

SEPTEMBER								
SNM	T	W	T	F	S			
松游	1	2,	उ	4	5			
67	(8)	9	10	11	12.			
13 14	15	16	17	18	19			
20 21	22	23	24	25	26			
27 28	29	30						

- 1. The first day of the **month** is Tuesday. The second day of the month is Wednesday. The third day of the month is Thursday; the fourth day of the month is \_\_?\_\_.
- 2. Which day of the week is the ninth? the twentieth? the eleventh? the sixteenth?
- 3. Which days of the month are ringed?
- 4. The 7th, \_\_?\_\_, \_\_?\_\_ and \_\_?\_\_, days of the month are Mondays.
- 5. If Tuesday is the fifteenth day of the month, which day of the month is Wednesday of that week?
- 6. Make up questions about the month in which you are studying this page. Perhaps you can make up better questions than ours. Write them out and have your neighbor answer them.
- 7. January is the first month of the year. Name the third, fifth, sixth, and tenth months of the year.
- **8.** Write the word to complete this sentence: My birthday is in the \_\_\_? month of the year.

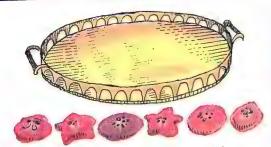
## Using sets and numbers



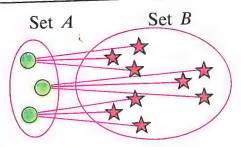
- 1. There are 6 children on the school bus and 7 children waiting to be picked up. Are there more children on the bus or more children waiting?
- 2. The bus passed 5 horses and 6 cows grazing in a field. Are there more horses or cows in the field?
- 3. Lisa counted the children on the school bus. She said, "two tens and seven ones." Write a two digit numeral for this number of children.
- 4. There are 3 cars in front of the school bus and 4 cars behind it. Are there more cars in front of or behind the school bus?
- **5.** There are 14 girls and 13 boys on the bus. Are there more girls or boys on the school bus?
- **6.** Sammy counted one-ten-three cars on the road. Write a two digit numeral for this number of cars.
- 7. Three children got off the bus at Elm Street.

  Tommy was the first one off the bus. Chuck was second. Sandy was \_\_\_\_?

#### One-to-many matching



There is 1 tray for 6 cookies. One tray matches 6 cookies.

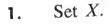


For each object in set A there are 3 objects in set B.

#### EXERCISES

How many objects in set Y are matched with each

object in set X?



Set Y.





2. Set X.

Set Y.





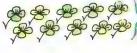
3. Set X.











Name the numbers to complete these sentences.

- 4. For every day there is a set of a hours.
- 5. For every cow there is a set of  $\boxed{z}$  legs.
- 6. For every week there is a set of c days.
- **7.** For every nickel there may be a set of  $\overline{u}$  pennies.
- 8. For every dime there may be a set of e pennies.

#### Dimes, nickels, pennies

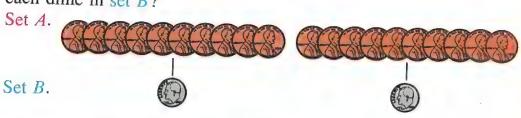


1. How many pennies in set A are matched with each nickel in set B?

Set A.

Set B.

2. How many pennies in set A are matched with each dime in set B?



Write the numerals to complete these sentences. You may use coins to help you.

- 3. One nickel matches 5 pennies, so 2 nickels match x pennies
- **4.** One dime matches 10 pennies, so 2 dimes match **c** pennies
- **5.** 3 nickels match **e** pennies
- 7. 4 nickels match **[Z]** pennies
- 9. 5 nickels match a pennies
- **6.** 3 dimes match **e** pennies
- **8.** 4 dimes match **r** pennies
- 10. 5 dimes match s pennies

The arrow on the number line shows the addition 10 + 1 = 11.



Use the number line to help you name the number of cents in:

- 11. 1 dime and 2 pennies
- 13. 1 dime and 4 pennies
- 15. 1 dime and 2 nickels

- 12. 2 dimes and 3 pennies
- 14. 2 dimes and 1 nickel
- 16. 2 nickels and 1 penny



#### Dimes, nickels, pennies







These are the coins.

A penny is worth 1 cent, a nickel is worth 5 cents, a dime is worth 10 cents.

These are their values.

#### EXERCISES

Name the value in cents.

- 1.

















7.







9.























6.



8.



10.







Write the numerals to complete these charts.

			Value in
•	Dimes	Pennies	cents
	3	4	34
11.	4	6	a
12.	5	2 -	υ
13.	6	3	c
14.	а	4	34
15.	и	6	26
16.	2	c	29
17	3	r	35

			Value in
	Nickels	Pennies	cents
18.	1	2	e
19.	1	3	r
20.	2	2	0
21.	2	4	z
<b>22</b> .	O	3	8
23.	n	z	9
24.	e	2	12
25.	2	S	14

#### The value of coins





The value of a quarter is 25 cents. We write this as 25¢.



The value of a half dollar is 50 cents. We write this as 50¢.

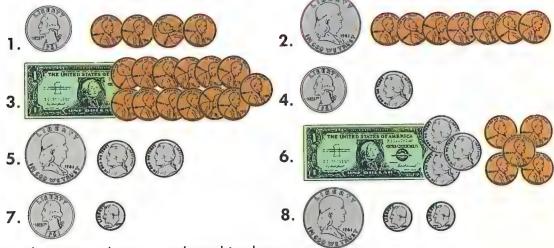


The value of a dollar is 100 cents.

We write this as \$1.00.

#### EXERCISES

Name the value in cents.

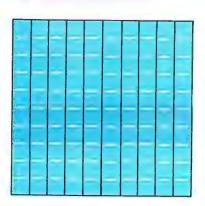


Write the numerals to complete this chart.

		Half					Value in
	Dollars	dollars	Quarters	Dimes	Nickels	Pennies	cents
	1			2		3	123
9.				1		7	a
10.	2		2			4	n
11.	1		3				c
12.	x	1	S				175
13.	,	а				n	53
14.			c			0	27
15.	e			r		S	112
16.		и	v		x		80

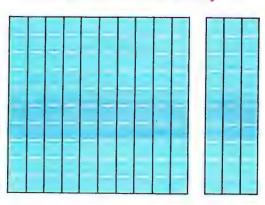
#### Hundreds and tens

The numeral in words is **one hundred**.



The 3-digit numeral is 100.

The numeral in words is **one hundred thirty**.



The 3-digit numeral is 130.

#### EXERCISES

Write the numerals to complete the tables.

1.	number of tens	10	20	40	70	80	30	50	60	90
	number of hundreds	1	2	a	n	c	0	e	v	z

2.	number of tens	10	12	21	24	r	S	79	83
	3-digit numeral	a	c	e	u	340	420	x	0
	numeral in words	z	e	S	n	u	r	n	a

Write the numerals to complete these patterns.

- **3.** 60, 70, 80, **a**, **n**, 110
- **5.** 160, 170, 180, **e**, **r**, **s**
- **7.** 120, 110, 100, **u**, **v**, 70
- **9.** 200, 190, **n**, **o**, **z**, 150
- 11. 300, **a**, 320, **c**, 340, **e**
- 13. 100, 110, **e**, **n**, **r**, **s**
- **15**. 600, 610, 620, **x**, **z**, 650

- **4.** 90, 100, **c**, **a**, 130, 140
- **6.** 200, 210, **v**, **z**, 240, 250
- 8. 140, 130, 120, **r**, **e**, 90
- **10.** 220, 210, **a**, **r**, **s**, 170
- **12.** 50, 40, **a**, 20, **c**, **e**
- 14. 30, 40, 50, **n**, **r**, **s**
- 16. 700, 690, 680, **x**, **z**, **u**

#### Hundreds and tens on the number line



80 90 100 110 120 130 140 150 160 170 180 190

> The name of the tens after 13 tens on the number line is 14 tens.

The 3-digit numeral for 14 tens is 140.

#### EXERCISES

Name as tens the next ten on the number line after:

- 1. 9 tens 2. 10 tens 3. 16 tens 4. 12 tens 5. 18 tens 6. 19 tens Write a 3-digit numeral for the number that is ten more than:
  - **7.** 90
- **8.** 120
- **9.** 230
- **10**. 450
- 11. 680
- **12.** 340

Name as tens the ten on the number line just before:

- **13.** 100
- 14. 120
- **15**. 170
- **16.** 210 **17.** 380
- **18**. 460

Write a 3-digit numeral for the number that is ten less than:

- **19.** 210
- **20**. 130
- **21.** 150
- **22.** 240
- **23**. 320
- **24.** 450

Name as tens the ten on the number line between:

**25.** 14 tens and 16 tens

**26.** 34 tens and 36 tens

**27**. 28 tens and 30 tens

**28.** 30 tens and 32 tens

**29**. 46 tens and 44 tens

**30.** 38 tens and 36 tens

Complete these number sentences. Write <, >, or = to replace each O.

- **31**. 390( )410
- )570 **32.** 520(
- **33.** 590( 600
- **34.** 8200

- **35.** 330( )303
- **36.** 420( )402

49

**37.** 420( 240

278

**38.** 380( )308

- )406 **39.** 64(
- 40. 39(
- **41**. 268(
- **42.** 3690
  - 409

840

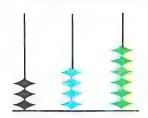
Name the tens to complete these number patterns.

- **43**. 80, 100, **a**, 140, **z**, 180, **c**, 220, 240, **o**
- **44.** 500, **u**, 460, 440, **v**, **x**, 380, 360, **z**, 320

#### Hundreds' place

Study these ways of showing the same number.

A. On the abacus



B. Using columns

Н	T	О
3	4	6

C. As an expanded numeral

$$300 + 40 + 6$$

D. Short form

346

E. In words

three hundred forty-six

The third place from the right is the hundreds' place.



Write the numerals to complete this chart.

	+	HTO	Expanded numeral	Short form
1.		a e c	n	0
2.	1 1 1	r s u	v	x
3.		a c e	<u>n</u>	0
4.	* * *	$n \mid o \mid r$	S	a

Name the number of hundreds in:

**5.** 348

**6.** 725

**7**. 168

**8.** 500

- **9.** 690
- **10.** 708
- 11. 56
- **12.** 1000

Name the sums.

#### Hundreds, tens, and ones



Write 3-digit numerals for these numbers.

1. two hundred fifty

2. seven hundred eight

3. four hundred twenty-nine

4. three hundred forty-seven

5. six hundred ninety-nine

6. nine hundred

Name the number:

- 7. just before 242
- 8. just before 250
- 9. just after 243

- **10.** just after 248
- 11. between 248 and 250
- 12. between 240 and 242

Name the hundred just before:

- **13.** 700
- 14, 200

**15**. 343

**16**. 651

Name the hundred just after:

**17.** 200

**18**. 800

**19.** 350

**20.** 680

Name the ten just before:

**21.** 760

**22.** 870

**23**. 925

**24**. 476

Name the ten just after:

- **25**. 230
- **26.** 540

**27.** 625

**28**. 386

Name the number between:

**29**. 486 and 488

**30**. 201 and 203

**31.** 319 and 321

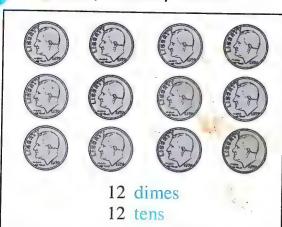
32. 299 and 301

Complete the number sentences. Write <, >, or = to replace the O.

- **33.** 324( )401
- **34.** 623( )631
- **35.** 100 + 99()200 1

- **36.** 316( )300 + 17 **37.** 200 + 40( )240 **38.** 300 + 40(
- **39.** 500 + 100 600 **40.** 461 164
- **41.** 465( )456
- **42.** 300 + 50()800 **43.** 521()
- **44.** 403(

#### Dollars, dimes, cents









1 dollar 2 dimes 100 + 20

#### EXERCISES

#### How many cents in:

- 1. 1 dollar
- 3. 2 dollars and 4 dimes

#### How many cents in:

- 5. 1 dollar, 2 dimes and 3 pennies
- 7. 1 dollar and 36 pennies

#### How many sets of 10 cents in:

- 9. 1 dollar and 3 dimes
- 11. 6 dollars and 5 dimes

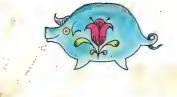
- 2. 1 dollar and 2 dimes
- 4. 7 dollars and 8 dimes
- 6. 1 dollar and 25 pennies
- **8.** 2 dollars and 36 pennies
- 10. 2 dollars and 4 dimes
- **12.** 3 dollars and 7 dimes

Write the numerals to complete this chart. We have done the first one.

		\$1	10¢	1¢	cents
		2	2	0	220
13.	<b>E E C C C C C C C C C C</b>	а	r	с	n
14.		e	υ	z	u
15.		s	z	n	r

#### Saving money

- 1. Erin opened her piggy bank. It contained 5 dollars and 3 dimes.
  - **a.** What was the value of this in dimes?
  - **b.** How many cents was this?



- 2. Sharon took 2 dollars, 4 dimes and 3 pennies to the bank. How many cents was this?
- 3. Arnie asked the bank teller to give him dimes for 6 dollars. How many dimes did he receive?



- 4. Lois had saved 17 dimes. Garry had saved 2 dollars and 1 dime. How many cents did each of them save?
- 5. In problem 4, who had saved more money, Lois or Garry?
- **6.** Jane had saved 340 pennies, Luke 3 dollars and 3 dimes, and Dickie 36 dimes.
  - a. Who had saved the most?
  - b. Who had saved the least?
- 7. Jacky could take his birthday present in one of these ways:
  - a. 2 dollars, 4 dimes, 3 pennies,
  - **b.** 245 cents,
  - **c.** 25 dimes.

Which should he choose?





#### Hundreds and thousands

100 200 300 400 500 600 700 800 900 1000 1100

10 hundreds = 1000 = one thousand

#### EXERCISES

Copy and complete this pattern of hundreds.

1. 100, 200, 300, **a**, **c**, **e**, **n**, **r**, 900

Name the hundred just after:

**2.** 300

**3**. 500

4. 600

**5.** 700

6. 900

Name the thousand just after:

**7.** 3000

**8.** 5000

**9.** 6000

**10.** 7000

11. 4000

**12.** 8000

**13**. 1000

14. 9000

Write 4-digit numerals for:

15. five thousand

16. six thousand

17. eight thousand

Name the number of hundreds in:

**18.** 4000

**19.** 6000

**20.** 8000

**21.** 2000

Name the number of thousands in:

**22.** 10 hundreds

**23.** 40 hundreds

24. 50 hundreds

**25**. 70 hundreds

**26.** 80 hundreds

**27**. 60 hundreds

Name the thousand just after:

**28.** 900

**29.** 2900

**30.** 3900

31. 4700

**32.** 3200

**33.** 4650

**34.** 5100

**35**. 1800

Name the thousand just before:

**36.** 1900

**37**. 2900

**38.** 3900

**39.** 4300

**40**. 4800

**41**. 6900

**42.** 3100

**43**. 8100

Name the thousand between:

**44.** 5 thousand and 7 thousand

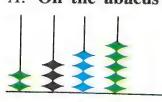
**45.** 9 thousand and 11 thousand

24 (twenty-four)

#### Thousands, hundreds, tens, ones



Study these ways of showing the same number.



A. On the abacus B. Using columns

C. As an expanded numeral

$$2000 + 300 + 40 + 5$$

#### D. Short form 2345

E. In words

two thousand three hundred forty-five

The fourth place from the right is the thousands' place.

#### EXERCISES

Write each numeral in two ways: as an expanded numeral, and in the short form.

Write these numerals in the short form.

- 5. Four thousand five hundred twenty-six
- 7. Eight thousand five

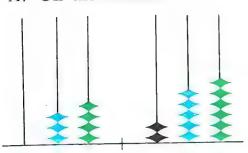
- 6. Six thousand nine hundred
- 8. Six thousand forty-five

Name the sums.

#### Tens of thousands

Study these ways of showing the same number.

A. On the abacus



Γhou	sands		Ones			
T	O	H	T	O		
3	4	2	5	6		

C. As an expanded numeral

$$30,000 + 4000 + 200 + 50 + 6$$

3. twenty-three thousand

7. sixty-eight thousand

5. eighty thousand

D. Short form 34,256

E. In words

Thirty-four thousand, two hundred fifty-six

We use a comma to mark off the thousands.

#### EXERCISES

Write the tens of thousands to complete this number line.

1. 
$$(10,000 20,000 30,000 40,000 50,000 a c)$$

Write a 5-digit numeral for

- 2. twelve thousand
- 4. fourteen thousand
- 6. ninety-seven thousand
- 8. twelve thousand, four hundred
- 9. fourteen thousand, five hundred twenty-six
- 10. forty-five thousand, two hundred fifty-nine

Write expanded numerals for these numbers.

- 11. 13,678
- **12.** 23,489
- **13**. 45,876
- **14**. 51,600

Write these numbers in the short form.

**15.** 
$$20,000 + 5000 + 400 + 20 + 7$$
 **16.**  $70,000 + 6000 + 900 + 50 + 6$ 

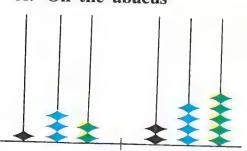
**17.** 
$$80,000 + 4000 + 100 + 70 + 8$$
 **18.**  $50,000 + 400 + 50 + 6$ 

#### Hundreds of thousands



Study these ways of showing the same number.

A. On the abacus



D. Short form 132,245

B. Using columns

Thousands			Ones			
H	T	O	H	T	0	
1	3	2	2	4	5	

C. As an expanded numeral 100,000 + 30,000 + 2000 + 2000 + 2000 + 5

E. In words one hundred thirty-two thousand, two hundred forty-five

#### EXERCISES

Write a 6-digit numeral for:

- 1. seven hundred thousand
- 2. nine hundred thousand

3. six hundred thousand

- 4. four hundred fifty thousand
- **5.** four hundred twenty-three thousand, six hundred forty-seven
- **6.** three hundred sixty-nine thousand, five hundred two
- 7. six hundred sixteen thousand, nine hundred fifty
- 8. nine hundred thousand, three hundred forty-two
- **9.** seven hundred forty thousand, six hundred seventy-five

Write expanded numerals for these numbers.

- **10.** 213,678
- 11. 423,489
- **12.** 545,876
- **13.** 651,600

Write these numbers in the short form.

**16.** 
$$700,000 + 4000 + 600 + 40 + 5$$

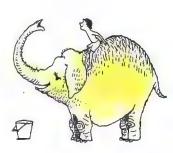
#### Interesting number facts

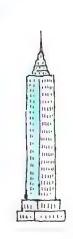
#### Did you know?

- 1. Your heart beats about 4000 times an hour.
- 2. You have more than 200 bones in your body.
- 3. This is a famous sight in Paris. Do you know its name? It is one thousand fifty-six feet high.
- 4. You probably weigh about 80 pounds.
- 5. Some elephants weigh more than 9000 pounds.
- 6. A car weighs about 3500 pounds.
- 7. There are 365 days in a year.
- 8. In 1965 there were 117 nations in the United Nations.
- 9. This airplane made a loud noise as it broke the sound barrier. It was flying more than seven hundred sixty-four miles an hour which is the speed of sound.
- 10. You probably have more than 5000 hairs on your head.
- 11. This is a famous sight in New York. Do you know its name? It is one thousand four hundred seventy-two feet high.

Can you find other interesting number facts like these? Make a list of them for your bulletin board.







#### Answer these questions.

- 1. What is the cardinal number of this set: {hat, coat, girl, book, milk carton}?
- 2. Show this set by listing the objects: {the first seven letters in the alphabet}
- 3. Name three subsets of the following set:  $\{1, 2, 3, 4, 5, 6\}$
- **4.** What do we call the symbols 1, 2, 3, 4, 5, 6, 7, 8, 9, 0?
- 5. Give 8 names for 10.
- 6. What is: a. the greatest counting number that can be named by writing a two-digit numeral?b. the least counting number that can be named by writing a three-digit numeral?

Write the numerals to complete this chart.

7.	н т о	Expanded numeral	Short form
*		v	e
8.	z a e	n	0

- 9. In the numeral 7684 which is digit b. the hundreds' digit c. the ones' digit d. the thousands' digit?
- 10. Write other numerals for a. one thousand forty-seven b. seven thousand, two hundred twenty-six c. nine thousand d. six hundred five.

### LOOKING BACK

A set is a collection or group. We can tell which things are in a set in two ways:

- 1. Listing the objects, for example, {spring, summer, fall, winter}
- 2. Describing the objects, for example, {the four seasons in the year}.
- 1. List the objects in {the last three letters of the alphabet}.
- 2. Describe the objects in {a, b, c, d}.

Two sets that can be matched one-to-one contain the same number of objects.

3. How many objects are there in a set which contains the same number of objects as a dog has legs?

Each number has many names. For example, 7 may be written as 6 + 1, 5 + 2, seven, or 9 - 2.

4. Write five other names for 4.

The number three hundred forty-seven may be shown as a three digit numeral, on an abacus, or as a sum of hundreds, tens, and ones.

5. Show the number two hundred fifty-nine in three different ways.

One number can be greater than, less than, or equal to another number.

6. Complete these sentences. Write <, >, or = to replace each ○.

**a.** 17 ( ) 37

**b.** 403 43

**c.** 894 270

#### EXTRA FOR EXPERTS



- 1. What is the greatest counting number that can be shown with five digits?
- 2. What is the least counting number that can be shown with five digits?
- 3. Write these numerals in two other ways.

**a.** 45

**b.** 64,389

**c.** 2681

Just for Fun

4. Which number is shown here?







Make up other puzzles like this one. Have your neighbor solve them.

- 5. Play the game, I am thinking of a number. Think of a number and then have other students in your class try to name the number you are thinking of. Use clues like these: "I am thinking of a number. It is less than 10 but greater than 5. One name for it has five letters. Its numeral is written with two straight strokes." The winner is the one who names the number first. He names the next number.
- 6. Play the game, Give me another name for this number. Give one name for a number to other students in your class. Then have them give you other names for the same number. The winner is the one who gives the most names.

# CHAPTER 2 • Addition-Subtraction Facts to 18

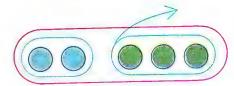




Our picture shows a set of 3 joined with a set of 2 to form a set of 5.

Three plus two equals five.

$$3 + 2 = 5$$



Our picture shows a set of 3 taken from a set of 5 to leave a set of 2.

Five minus three equals two.

$$5 - 3 = 2$$

#### EXERCISES

Set A and set B form set C. Complete the chart.

3 8 cardinal number of  $A \mid 3$ 6  $\boldsymbol{a}$ 1. 3 6 4 cardinal number of B5 3 r n10 10 cardinal number of C 10 9 S u

Set X take away set Y leaves set Z. Complete the chart.

cardinal number of X6 4 10 8 n 8 Z 2. 3 cardinal number of Y 2 5 S 6 4 e  $\boldsymbol{a}$ 4 10 cardinal number of Z3 v

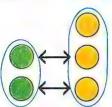
Complete the sentences.

- 3. When we join sets, we think about \_\_?
- 4. When we separate sets, we think about ?\_\_ numbers.

#### **Comparing sets**



Set A Set B



Set B contains one unmatched object. Set B contains one more object than set A. Three minus two equals one. 3 - 2 = 1 The difference between 3 and 2 is 1.

#### EXERCISES

Write the numeral to replace each  $\square$ .

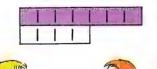
1. The difference between the number of children and the number of chairs is [a].



2. A set of 10 is  $\overline{a}$  larger than a set of 6. A set of 6 is  $\overline{r}$  smaller than a set of 10. The difference between 10 and 6 is  $\overline{e}$ .  $10 - \overline{v} = 6$ ,  $6 + \overline{r} = 10$ .



- 3. The difference between the longer rod and the shorter rod is  $\overline{u}$ .
- 4. A set of 7 is a larger than a set of 4.
  A set of 4 is m smaller than a set of 7.
  The difference between 7 and 4 is c.
  7 4 = s, 4 + c = 7.

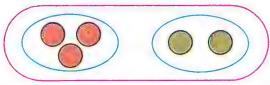


**5.** The girl has **C** more flowers than the boy.



6. To name the \_\_?\_ between the number of objects in two sets, subtract the cardinal number of the smaller set from the cardinal number of the larger set.

### Equations and working forms



5 and 3 + 2 are two names for the same number.

5 and + 2 are two names for the same number.

$$3 + 2 = 5$$
 is an equation.

$$\frac{3}{+2}$$
 is a working form.

3 and 2 are addends, 5 is the sum.

#### EXERCISES

Write the numerals that name the same number as the numeral on the left.

1. 4: 
$$3 + 1$$
,  $7 - 3$ ,  $2 + 1 + 1$ ,  $40$ ,  $4 + 0$ 

**2.** 5: 
$$3 + 2$$
,  $2 + 3$ ,  $1 + 1 + 1$ ,  $0 + 5$ ,  $41$ 

3. 6: 
$$4-2$$
,  $4+2$ ,  $7-2$ , six, five plus one

Which of these equations are true?

**5.** 
$$3 + 2 = 6$$

**5.** 
$$3 + 2 = 6$$
 **6.**  $9 - 3 = 4 + 2$  **7.**  $10 + 2 = 20$ 

7. 
$$10 + 2 = 20$$

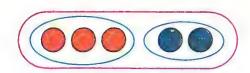
8. 
$$6 + 0 = 6$$

**8.** 
$$6 + 0 = 6$$
 **9.**  $4 + 1 = 3 + 3$  **10.**  $34 = 30 + 4$ 

10. 
$$34 = 30 + 4$$

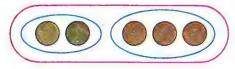
Complete these examples.

Write the equations for these working forms.



$$3 + 2 = 5$$

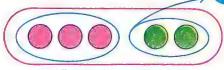
Addend plus addend equals sum.



$$2 + 3 = 5$$

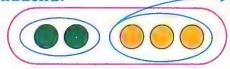
Addend plus addend equals sum.





$$5 - 2 = 3$$

Sum minus addend equals addend.



$$5 - 3 = 2$$

Sum minus addend equals addend.

These four equations make up a family of facts.

#### EXERCISES

Complete these equations.

1. Since 
$$3 + 6 = 9$$
,  $9 - 6 = |\overline{u}|$ 

4. Since 
$$4 + 5 = 9$$
,  $9 - 5 = |\overline{x}|$ 

2. Since 
$$4 + 6 = 10$$
,  $10 - 6 = |v|$ 

**5.** Since 
$$2 + 8 = 10$$
, **6.** Since  $11 - 5 = 6$ ,  $10 - 8 = \boxed{z}$ 

3. Since 
$$9 - 7 = 2$$
,  $9 - 2 = [a]$ 

**6.** Since 
$$11 - 5 = 6$$
  $11 - 6 = |c|$ 

Copy and complete these equations.

7. 
$$3 + a = 9$$
  $e + 3 = 9$   $9 - c = 3$   $9 - 3 = n$ 

$$e + 3 = 9$$

$$9 - |\overline{c}| =$$

$$9-3=n$$

**8.** 
$$4 + \mathbf{r} = 10$$
 **s**  $+ 4 = 10$   $10 - 4 = \mathbf{u}$   $10 - \mathbf{v} = 4$ 

$$s + 4 = 10$$

$$10 - 4 = \overline{u}$$

$$10 - v = 4$$

**9.** 
$$5 + x = 9$$
  $z + 5 = 9$   $9 - 5 = a$ 

$$z + 5 = 9$$

$$9-5=\overline{a}$$

$$9 - [e] = 5$$

10. 
$$4 + \overline{n} = 7$$
  $\overline{r} + 4 = 7$   $7 - \overline{s} = 4$   $7 - 4 = \overline{u}$ 

$$|r| + 4 = 7$$

$$7 - |s| = 4$$

$$7-4=\overline{u}$$

Write 3 equations to complete the family for each of these equations.

11. 
$$4 + 2 = 6$$

12. 
$$2 + 5 = 7$$

13. 
$$3 + 7 = 10$$

14. 
$$2 + 7 = 9$$

**15.** 
$$5 + 1 = 6$$

**16.** 
$$3 + 5 = 8$$

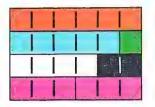
Complete each sentence.

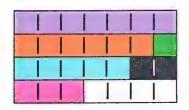
- 17. The order of the addends does not affect the \_\_?\_.
- **18.** Subtraction is the opposite of \_\_?\_.

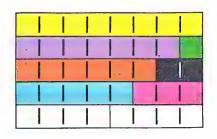


#### Sums of 6, 7, and 8

Study these colored rods.







#### EXERCISES

Use the colored rods to help you solve these equations.

1. 
$$4 + 2 = a$$

2. 
$$5 + 2 = n$$

3. 
$$6 + 2 = c$$

4. 
$$3 + 4 = e$$

5. 
$$8 - 1 = s$$

6. 
$$8 + 0 = n$$

7. 
$$5 + c = 8$$

8. 
$$2 + 6 = v$$

9. 
$$7 + z = 8$$

10. 
$$7 - 2 = n$$

11. 6 - 1 = 
$$n$$

12. 
$$4 + 4 = v$$

13. 
$$5 + 3 = 0$$

14. 
$$8 - 2 = u$$

15. 
$$7 - 7 = s$$

16. 
$$7 - 3 = c$$

17. 
$$3 + a = 7$$

18. 
$$6 - 0 = n$$

Name the sums.

Name the missing addends.

34.

$$\frac{-4}{e}$$

8

35. 
$$\frac{6}{-a}$$

$$\frac{-\cancel{e}}{5}$$

$$\frac{- \boxed{n}}{3}$$

|a|

$$\frac{6}{-\cancel{x}}$$

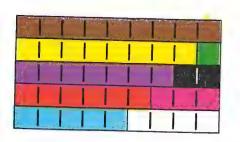
$$\frac{-n}{n}$$

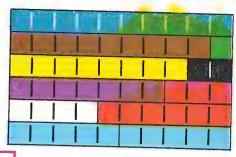
$$\frac{-x}{5}$$

### Sums of 9 and 10



Study these colored rods.





#### EXERCISES

Use the colored rods to help you solve these equations.

1. 
$$7 + 2 = a$$

**2.** 
$$7 + 3 = c$$

3. 
$$6 + \overline{n} = 10$$

4. 
$$1 + |e| = 9$$

5. 
$$6 + |r| = 9$$

4. 
$$1 + \boxed{e} = 9$$
5.  $6 + \boxed{r} = 9$ 
6.  $10 - 4 = \boxed{s}$ 

7. 
$$8 + 0 = 10$$

8. 
$$7 + |\overline{u}| = 10$$

7. 
$$8 + \boxed{0} = 10$$
 8.  $7 + \boxed{u} = 10$  9.  $5 + \boxed{v} = 10$ 

10. 
$$5 + |x| = 9$$

11. 
$$8 + |n| = 9$$

10. 
$$5 + \bar{x} = 9$$
 11.  $8 + \bar{n} = 9$  12.  $8 + 1 = \bar{z}$ 

13. 
$$4 + [e] = 0$$

14. 
$$3 + |r| = 10$$

**13.** 
$$4 + \mathbf{e} = 9$$
 **14.**  $3 + \mathbf{r} = 10$  **15.**  $9 - 9 = \mathbf{a}$ 

Name the sums.

Name the missing addends.

23. 
$$\frac{7}{\frac{+ a}{0}}$$

$$\begin{array}{r}
7 \\
+ \boxed{c} \\
\hline
10
\end{array}$$

7. 
$$\frac{n}{+5}$$

29. 
$$\frac{5}{10}$$

lame the missing addends or sums.

30. 
$$2$$
 31.  $6$  32.  $7$  33.  $9$  34.  $4$  35.  $8$  36.  $+ \overline{u}$ 

$$\begin{array}{c}
6 \\
+ \boxed{r} \\
\hline
10
\end{array}$$

$$\frac{7}{+3}$$

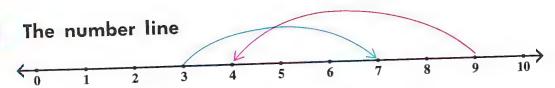
$$\frac{+ |\vec{a}|}{9} = \frac{+ |\vec{r}|}{10} = \frac{+ 3}{|\vec{c}|} = \frac{9}{9} = \frac{34.}{|\vec{n}|} = \frac{4}{10} = \frac{36.}{|\vec{v}|} = \frac{10}{9}$$

**36.** 
$$v$$
 + 3

**37.** 
$$9 + 0 = \mathbf{z}$$
 **38.**  $9 - \mathbf{a} = 9$  **39.**  $\mathbf{s} - 4 = 6$ 

38. 
$$9 - a = 9$$

**39.** 
$$s - 4 = 6$$

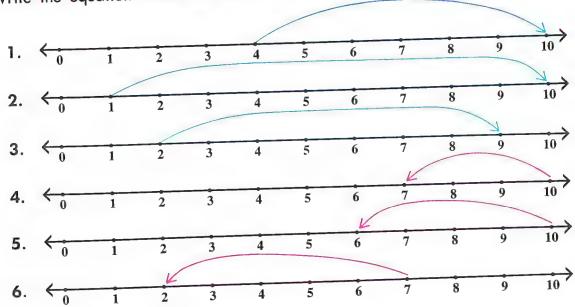


The blue arrow shows the equation 3 + 4 = 7.

The red arrow shows the equation 9 - 5 = 4.

### EXERCISES

Write the equation shown on each number line.



Use the number line to help you write an equation, then solve the equation.

	Start	Move right	Stop
7.	3	6	а
8.	2	7	С
9.	3	5	e
10.	5	4	n
11.	6	2	0
12.	3	r	10
13.	4	S	9

	Start	Move left	Stop
14.	6	3	c
15.	7	2	e
16.	8	1	$\boldsymbol{n}$
17.	10	2	0
18.	9	4	r
19.	4	S	2
20.	10	u	1

## Number patterns and number pairs



The arrows show the additions 1 + 3 = 4, 2 + 3 = 5, 3 + 3 = 6, and so on. These additions may be set up to form a number pattern like this:

+	1	2	3	4	5	6	7
3	4	5	6	7	8	9	10

They may also be set up as number pairs like these: (1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9), (7, 10). The rule to name the second number from the first number in each pair is: Add 3.

#### EXERCISES

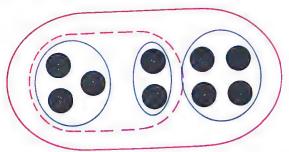
Complete these number patterns.

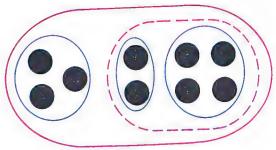
Complete these sets of number pairs. Use the first pairs in each set to find the rule. Then write the missing numerals.

- **5.**  $\{(3, 4), (4, 5), (5, \mathbf{a}), (6, \mathbf{c}), (7, \mathbf{n}), (8, \mathbf{x})\}$
- **6.**  $\{(1, 5), (2, 6), (3, \mathbf{C}), (4, \mathbf{S}), (5, \mathbf{U}), (6, \mathbf{n})\}$
- **7.**  $\{(2, 5), (3, 6), (4, \overline{v}), (5, \overline{x}), (6, \overline{z}), (7, \overline{a})\}$
- **8.**  $\{(6,3), (5,2), (10, \mathbf{n}), (9, \mathbf{s}), (8, \mathbf{u}), (7, \mathbf{v})\}$
- **9.**  $\{(10,8), (7,5), (6, \mathbf{r}), (8, \mathbf{s}), (5, \mathbf{e}), (4, \mathbf{n})\}$
- **10.**  $\{(10,6), (5,1), (7, \mathbf{a}), (8, \mathbf{v}), (6, \mathbf{r}), (7, \mathbf{u})\}$
- 11.  $\{(1,1), (2,2), (3, 0), (4, 0), (6, 0), (8, n)\}$

## Grouping addends

Can you add three addends at the same time?





$$(3 + 2) + 4 = 3 + (2 + 4)$$
  
 $5 + 4 = 3 + 6$ 

The addends are grouped two at a time. The way the addends are grouped does not affect the sum. This is the Grouping Rule. We use ( ) to show the Grouping Rule.

#### EXERCISES

Check that these equations are true. Name the sums.

1. 
$$(3+2)+5=3+(2+5)$$

**2.** 
$$(3+1)+5=3+(1+5)$$

**1.** 
$$(3+2)+3=3+(2+4)$$
  
**3.**  $(4+2)+4=4+(2+4)$   
**4.**  $(6+2)+1=6+(2+1)$ 

**4.** 
$$(6+2)+1=6+(2+1)$$

**5.** 
$$(3+1)+3=3+(1+3)$$

**5.** 
$$(3+1)+3=3+(1+3)$$
 **6.**  $(2+2)+5=2+(2+5)$ 

7. 
$$6 + (2 + 2) = (6 + 2) + 2$$

**5.** 
$$(3+1)+3=3+(1+3)$$
  
**7.**  $6+(2+2)=(6+2)+2$   
**8.**  $(3+2)+4=3+(2+4)$ 

9. 
$$(3+3)+4=3+(3+4)$$

7. 
$$6 + (2 + 2) = (6 + 2) + 2$$
  
9.  $(3 + 3) + 4 = 3 + (3 + 4)$   
10.  $(2 + 5) + 3 = 2 + (5 + 3)$ 

Name the sums by adding down, then by adding up.

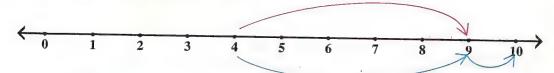
Complete the sentence.

18. The Grouping Rule says that the way the ? are grouped does not affect the sum.

### The grouping rule



We may show the grouping of addends on the number line.



The red arrow shows the addition 4 + 5 = 9. The blue arrows show how to use the addition 4 + 5 = 9 to find the sum 4 + 6. The equations would be: 4 + 6 = 4 + (5 + 1)= (4 + 5) + 1

$$= 9 + 1$$
  
= 10

#### EXERCISES

Name the three addends and the sum shown on each of these number lines.





5 -7. The addition in exercise 1 may be written as 3 + 3 + 4 = 10. It may also be written as 6 + 4 = 10 or 3 + 7 = 10. For exercises 2, 3, and 4, write two equations with two addends and a sum.

Solve these equations.

**8.** 
$$3 + 4 = 7$$
, so  $3 + \boxed{a} + 1 = 8$ 

9. 
$$6+2=8$$
, so  $6+\boxed{c}+1=9$ 

**10.** 
$$2 + 5 = 7$$
, so  $2 + \boxed{e} + 1 = 8$ 

11. 
$$5 + 3 = 8$$
, so  $5 + \mathbf{m} + 1 = 9$ 

**12.** 
$$3 + 6 = 9$$
, so  $3 + 6 + \boxed{0} = 10$ 

13. 
$$4+4=8$$
, so  $4+4+|r|=9$ 

**14.** 
$$2 + 7 = 9$$
, so  $2 + 7 + \boxed{s} = 10$ 

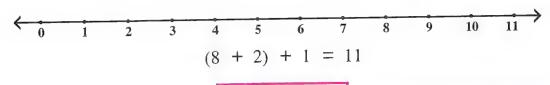
**15.** 
$$7 + 1 = 8$$
, so  $7 + 1 + \overline{u} = 9$ 

#### The grouping rule

It is often useful to rename two addends as three addends and apply the grouping rule. Study this example.

$$8 + 3 = 8 + (2 + 1)$$
  
=  $(8 + 2) + 1$   
=  $10 + 1$   
=  $11$ 

We may show this grouping on the number line.



#### EXERCISES

The equations for the additions on the number line above would be 8 + 2 + 1 = 11 or 8 + 3 = 11. For each of these additions write two equations, one with three addends and one with two addends.



Copy and complete these equations.

5. 
$$7 + 4 = 7 + (3 + \overline{a})$$
  
=  $(7 + 3) + \overline{a}$   
=  $10 + \overline{a}$   
=  $\overline{x}$ 

6. 
$$8 + 4 = 8 + (2 + e)$$
  
=  $(8 + 2) + e$   
=  $10 + e$   
=  $z$ 

7. 
$$6 + 5 = 6 + (4 + \overline{u})$$
  
=  $(6 + 4) + \overline{u}$   
=  $10 + \overline{u}$   
=  $\overline{s}$ 

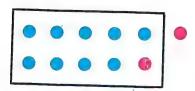
8. 
$$7 + 5 = 7 + (a + c)$$
  
=  $(7 + a) + c$   
=  $10 + c$   
=  $n$ 

### Sums of 11 and 12



Study this pattern.

9 + 2 = 11

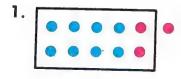


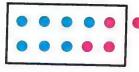
$$2 + 9 = 11$$
  $11 - 9 = 2$   $11 - 2 = 9$ 

Read these equations.

$$9 + 2 = 9 + (1 + 1)$$
  
=  $(9 + 1) + 1$   
=  $10 + 1$ 

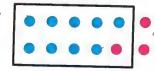
Write an addition and a subtraction equation for each of these patterns.



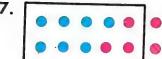


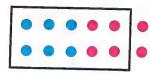


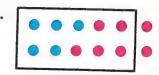












Name the sums.

11. 8 12. 9 13. 6 14. 3 15. 6 16. 7

+ 6

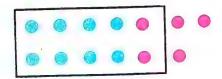
lame the missing addends.

17. 5 18. 
$$a$$
 19.  $c$  20. 7 21.  $e$  22. 8  $+ u$   $+ 2$   $+ 2$   $+ m$   $+ 4$   $+ o$ 

23. S

### Sums of 13, 14, and 15

Study this pattern.



Read these equations.

$$8 + 5 = 8 + (2 + 3)$$
  
=  $(8 + 2) + 3$   
=  $10 + 3$   
=  $13$ 

$$8 + 5 = 13,$$

$$5 + 8 = 13$$

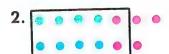
$$5 + 8 = 13$$
,  $13 - 5 = 8$ ,  $13 - 8 = 5$ 

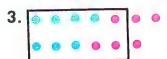
$$13 - 8 = 5$$

#### EXERCISES

Write an addition and a subtraction equation for each of these patterns.



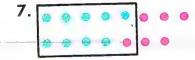




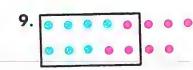












Name the sums.

10. 
$$6 + 7 = a$$

11. 
$$6 + 8 = C$$

12. 
$$6 + 9 = e$$

13. 
$$7 + 7 = 0$$

14. 
$$7 + 8 = r$$

15. 
$$9 + 6 = s$$

Solve the equations.

**16.** 
$$6 + 6 = 12$$
, so  $6 + 7 = \overline{a}$ 

**18.** 
$$6 + 7 = 13$$
, so  $6 + 8 = \boxed{c}$ 

**20.** 
$$6 + 8 = 14$$
, so  $6 + 9 = e$ 

**22.** 
$$5 + 8 = 13$$
, so  $5 + 9 = 5$ 

17. 
$$5 + 5 = 10$$
, so  $5 + 6 = \overline{u}$ 

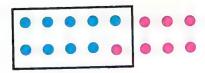
**19.** 
$$5 + 6 = 11$$
, so  $5 + 7 = \mathbf{r}$ 

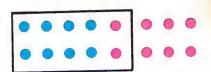
**21.** 
$$5 + 7 = 12$$
, so  $5 + 8 = 0$ 

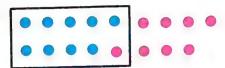
**23.** 
$$5 + 9 = 14$$
, so  $5 + 10 = \mathbf{Z}$ 



Write an addition and a subtraction equation for each of these patterns.









Name the sums.

Name the missing addends.

$$9 + |a| = 18$$

10. 
$$8 + \boxed{c} = 16$$

9. 
$$8 + \boxed{a} = 18$$
 10.  $8 + \boxed{c} = 16$  11.  $9 + \boxed{c} = 18$ 

Name the sums or missing addends.

$$\frac{+ n}{18}$$

$$\frac{+8}{|x|}$$

$$\frac{+9}{r}$$

$$\frac{r}{+7}$$

Solve the equations.

19. 
$$8 + 8 = 16$$
, so  $16 - 8 =$ 

**19.** 
$$8 + 8 = 16$$
, so  $16 - 8 = \boxed{a}$  **20.**  $7 + 9 = 16$ , so  $16 - 9 = \boxed{c}$ 

**21.** 
$$8 + 9 = 17$$
, so  $17 - 9 = \boxed{c}$ 

**22.** 
$$9 + 7 = 16$$
, so  $16 - 7 = \mathbf{n}$ 

**23.** 
$$9 + 9 = 18$$
, so  $18 - 9 = \boxed{x}$ 

**23.** 
$$9 + 9 = 18$$
, so  $18 - 9 = x$  **24.**  $9 + 8 = 17$ , so  $17 - 8 = r$ 

Name the sums.

**25.** 
$$6 + 3 + 7$$

### Using sets and numbers

Ben uses a 5-step method for solving problems.

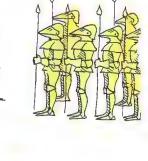
- Step 1. He asks "What are the sets in the problem?
- Step 2. He asks "What is happening to these sets?"

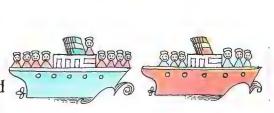
  Are the sets being joined, separated or compared?
- Step 3. He uses the cardinal numbers of the sets to write an equation.
- Step 4. He solves his equation.
- Step 5. He now uses the number he has found to answer the question about the sets in the problem.

#### EXERCISES

Use the 5-step method to solve these problems.

- 1. Tom has 8 knights and his brother has 6. How many more knights has Tom?
- 2. Grace played three games of ring toss. She scored 4, 5, and 6. What was her total score?
- 3. A kite string costs 8¢ and a kite costs 15¢. How much more does the kite cost?
- 4. It took Jimmy 9 minutes to walk to school and 5 minutes to ride home. How much longer did it take him to walk?
- **5.** In one boat there were 9 sailors. In the other boat there were 6. How many sailors were there in all?
- 6. Mary saved 17 cents, and Joan saved 9 cents. How much more money did Mary save than Joan?





### The football game



- 1. If there are 7 players on the red team and 6 players on the blue team, how many players are there in all?
- 2. In problem 1, which team has more players? How many more?
- 3. Three more boys want to play. a. How many are put on each team to make the teams even? b. How many boys are there now in all?
- **4.** The red team has a score of 9. The blue team has a score of 6. Which team is winning?
- 5. There are 4 girls cheering for the red team and 6 girls cheering for the blue team. How many more girls are cheering for the blue team?
- 6. How many cheerleaders are there all together?
- **7.** Five boys had to go home for supper. How many boys were left?
- 8. The final score was 13 to 9. By how many points did the blue team win?

#### Addition-subtraction

$$6 + 2 = \mathbf{a}$$
  
 $2 + 6 = \mathbf{c}$   
 $\mathbf{e} - 6 = 2$   
 $\mathbf{n} - 2 = 6$ 

In each of these equations you have two addends and are looking for the sum.

These are addition equations.

$$8 - 2 = 0$$
  
 $8 - r = 2$   
 $2 + s = 8$   
 $u + 2 = 8$ 

In each of these equations you have the sum and one addend and are looking for the missing addend.

These are subtraction equations.

#### EXERCISES

Complete the chart.

1.	addend	3	2	3	4	2	8	6	9	8	n	0	e
	addend	6	6	7	8	9	e	и	v	z	2	6	3
	sum	9	a	r	c	S	15	14	13	14	11	10	12

Solve these equations, then write three more equations to go with each given equation.

$$2.8 + 3 = a$$

**2.** 
$$8 + 3 = a$$
 **3.**  $6 + 7 = r$ 

4. 
$$9 + \boxed{c} = 12$$

**5.** 
$$8 + \boxed{s} = 13$$
 **6.**  $7 + 9 = \boxed{n}$ 

6. 
$$7 + 9 = n$$

7. 
$$4 + 7 = c$$

Name the sums or missing addends.

8. 
$$8 + 5 = a$$

**8.** 
$$8 + 5 = \overline{a}$$
 **9.**  $8 + 6 = \overline{0}$ 

10. 
$$9 + 7 = r$$

11. 
$$e - 4 = 5$$

12. 
$$s - 3 = 10$$

13. 
$$u - 7 = 7$$

14. 
$$3 + \bar{a} = 8$$

15. 
$$7 + r = 12$$

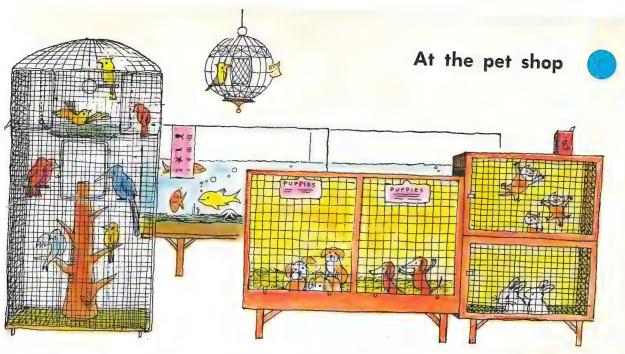
**16.** 
$$\mathbf{0} + 6 = 10$$

17. 
$$15 - 8 = \overline{u}$$

18. 
$$17 - \mathbf{z} = 9$$

19. 
$$12 - \overline{n} = 8$$

Name the sums.



- 1. Donna and Julie went to the pet shop. They saw 3 kittens in one basket, and 5 in another. How many kittens were in both baskets?
- 2. There were 14 angel fish and 8 swordtails in a tank. How many more angel fish were there than swordtails?
- **3.** In one cage there were 7 blue parakeets and 6 green ones. How many parakeets were in the cage?
- **4.** Julie counted 5 bunnies in a box, and 7 baby hamsters in another box. How many animals were there in all?
- **5.** There were 9 brown puppies and 4 white ones. How many more brown puppies were there?
- 5. On one side of the pet shop there were 9 tanks of tropical fish, and 6 tanks on the other side. How many tanks were there all together?
- 7. In a big box there were 7 small green turtles and 8 large ones. How many turtles were there?

### Subtraction



Rudy has forgotten that 5 + 8 = 13, so he cannot solve the equation 13 - 8 = a.



Mary says, "Try this method."

Mary shows her method on the number line like this.



#### EXERCISES

Copy and complete these equations.

1. 
$$14 - 9 = (14 - 4) - 5$$
  
=  $10 - 5$   
=  $\overline{a}$ 

3. 
$$16 - 7 = (16 - \overline{r}) - \overline{s}$$
  
=  $10 - \overline{s}$   
=  $\overline{v}$ 

2. 
$$17 - 8 = (17 - \boxed{c}) - \boxed{n}$$
  
=  $10 - \boxed{n}$   
=  $\boxed{o}$ 

4. 
$$13 - 5 = (13 - x) - z$$
  
=  $10 - z$   
=  $c$ 

Draw number lines to show these subtractions.

**5.** 
$$15 - 9 = 6$$

**5.** 
$$15 - 9 = 6$$
 **6.**  $16 - 7 = 9$ 

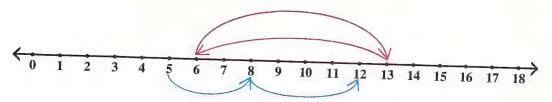
**7.** 
$$14 - 8 = 6$$

Name the differences.

Complete these number patterns.

### Using the number line





The red arrows show the equations 6 + 7 = 13 and 13 - 7 = 6. The blue arrows show the equation 5 + 3 + 4 = 12.

#### EXERCISES

Draw a number line and use it to help you name the sums or missing addends.

1. 
$$9 + a = 18$$

**2.** 
$$13 - \boxed{c} = 6$$

3. 
$$13 + 5 = [e]$$

4. 
$$14 - x = 8$$

5. 
$$10 - \mathbf{r} = 3$$

6. 
$$s - 2 = 11$$

7. 
$$a - 5 = 9$$

8. 
$$7 + \boxed{c} = 15$$

9. 
$$6 + 12 = e$$

10. 
$$n - 4 = 9$$

11. 
$$13 + 4 = \bar{x}$$

12. 
$$x + 6 = 8$$

**13.** 
$$4 + 3 + 0 = 12$$

14. 
$$3 + 1 + r = 10$$

**13.** 
$$4 + 3 + \boxed{0} = 12$$
 **14.**  $3 + 1 + \boxed{r} = 10$  **15.**  $4 + 7 + \boxed{s} = 16$ 

16. 
$$6 + 2 + \overline{u} = 15$$

**19.**  $3 + 4 + \boxed{c} = 13$ 

**17.** 
$$3 + 2 + \overline{v} = 12$$
 **18.**  $6 + 1 + \overline{x} = 10$ 

**20.** 
$$2 + 3 + \overline{a} = 12$$
 **21.**  $2 + 5 + 3 = \overline{z}$ 

Jse a number line to help you name the sums.

**22.** 
$$3 + 5 + 6$$

**23.** 
$$6 + 2 + 7$$

**25.** 
$$2 + 7 + 2$$

**27.** 
$$4 + 2 + 5$$

**28.** 
$$6 + 3 + 2$$

**30.** 
$$5 + 2 + 5$$

$$31. 2 + 3 + 8$$

**32.** 
$$4 + 3 + 4$$

**33.** 
$$5 + 2 + 1$$

lame the missing addends.

3 **37**. 6 **38**. 5 
$$\frac{1}{2}$$

$$\frac{9}{12}$$

#### The number line



On the number line we see that 6 + 5 comes just after 10 and just before 12; 6 + 5 is between 10 and 12.

#### EXERCISES

Name the number on the number line that comes just after:

Name the number on the number line that comes just before:

Name the number on the number line that comes between:

**25.** 
$$3 + 4$$
 and  $4 + 5$  **26.**  $6 + 5$  and  $9 + 4$  **27.**  $4 + 6$  and  $8 + 4$ 

**28.** 
$$5 + 4$$
 and  $6 + 5$  **29.**  $5 + 7$  and  $7 + 7$  **30.**  $8 + 2$  and  $5 + 3$ 

**31.** 
$$9 + 3$$
 and  $7 + 7$  **32.**  $7 + 4$  and  $5 + 4$  **33.**  $6 + 9$  and  $8 + 5$ 

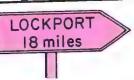
Answer the questions.

- 34. The Lake's car had gone 18 miles. The Gould's car had gone 16 miles. The Marin's car had gone farther than the Gould's car but not as far as the Lake's car. What color is
  - a. the Lake's car, b. the Gould's car,
  - c. the Marin's car?

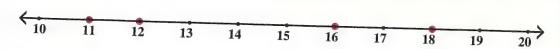








#### The number line



Eleven is less than twelve.

Eighteen is greater than sixteen.

11 < 12

18 > 16

#### EXERCISES

Complete these number sentences. Write <, >, or = to replace each  $\bigcirc$ .

1. 
$$4 + 6 \bigcirc 9$$

**4.** 
$$3 + 5 \bigcirc 7$$

**7.** 
$$3 + 5 \bigcirc 5$$

10. 
$$4 + 3 () 5 + 2$$

12. 
$$6 + 6 \bigcirc 5 + 7$$

**14.** 
$$13 - 7 \bigcirc 8 - 1$$

**16.** 9 + 8 
$$\bigcirc$$
 10 + 6

18. 
$$3 + 6 \bigcirc 4 + 5$$

**20.** 
$$2 + 3 \bigcirc 3 + 2$$

**2.** 
$$4 + 7 \bigcirc 10$$

**8.** 9 + 2 
$$\bigcirc$$
 10

3. 
$$4 + 7 \bigcirc 12$$

11. 
$$3 + 3 \bigcirc 3 + 4$$

**13.** 
$$5 + 6()5 + 7$$

15. 
$$5 + 4 \bigcirc 3 + 7$$

17. 
$$4 + 8 \bigcirc 10 + 3$$

19. 
$$6 - 2 \bigcirc 5 - 1$$

Complete these number sentences. Write < or > to replace each O.

**22.** 
$$7 > 6$$
, so  $7 + 2 \bigcirc 6 + 2$ 

**24.** 
$$7 + 5 > 6 + 5$$
, so  $7 \bigcirc 6$ 

**26.** 
$$2 < 7$$
, so  $12 \bigcirc 17$ 

**23.** 
$$8 > 7$$
, so  $8 + 4 \bigcirc 7 + 4$ 

**25.** 
$$5 + 4 < 8 + 4$$
, so  $5 \bigcirc 8$ 

Arrange the numbers in these sets in order with the least number first.

**28.** 
$$\{4+6, 4+3, 4+5, 4+4, 4+7\}$$

**29.** 
$$\{6 + 1, 6 + 0, 6 + 6, 6 + 5, 6 + 4\}$$

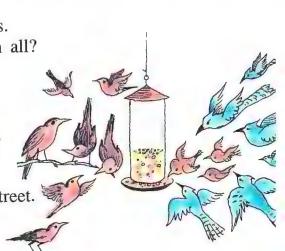
**30.** 
$$\{0 + 6, 7 + 0, 0 + 9, 8 + 0, 0 + 0\}$$

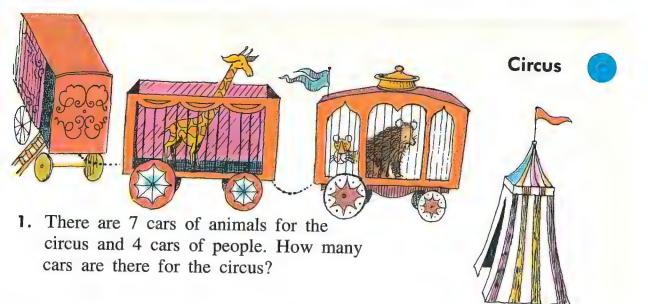
### Using sets and numbers

1. From a set of 14 take a set of 8. This leaves a set of  $\overline{a}$ .



- 2. Joining a set of 8 and a set of 6 forms a set of  $\boxed{n}$ .
- **3.** If you have 12 cents and spend 9 cents, how many cents do you have left?
- **4.** If you spend 9 cents and then 5 cents, how many cents do you spend in all?
- 5. At the circus there are 8 monkeys and 6 lions. How many animals are there in all?
- **6.** There are 9 bikes and 14 boys. How many boys do not have bikes?
- 7. You had 13 cents and lost 4 cents. How many cents do you have left?
- 8. You have 7 blue and 6 red marbles. How many marbles do you have in all?
- 9. There were 7 robins, 6 blue jays, and 3 wrens at the bird feeder. How many birds were there in all?
- 10. There were 10 houses on Green Street. If 7 are brown, how many are not brown?





- 2. An adult ticket for the side show costs 17¢. A child's ticket costs 8¢. What is the difference in the cost?
- **3.** There were 17 buckets of water. The elephants drank 9. How many buckets of water were left?
- 4. Don counted 2 elephants, 3 tigers and 7 horses in the circus parade. How many animals did Don see?
- **5.** Eight horses have riders. Seven horses do not have riders. How many horses are there in all?
- 5. A tiny car stopped and 7 clowns got out. 9 clowns got out of another car. How many clowns were there in all?
- '. There were 17 clowns in the red car.

  Nine clowns wore hats. How many did not wear hats?





### Building the addition-subtraction chart

David made a chart to show the sum when 1 is one of the addends. 
 +
 1
 2
 3
 4
 5
 6
 7
 8
 9

 1
 2
 3
 4
 5
 6
 7
 8
 9
 10

Then he made a chart to show the sum when 2 is one of the addends.

 +
 1
 2
 3
 4
 5
 6
 7
 8
 9

 2
 3
 4
 5
 6
 7
 8
 9
 10
 11

He put the charts together to form one chart.

**Columns** 

He built up his chart to show all the numbers from 1 to 9 as addends.

 +
 1
 2
 3
 4
 5
 6
 7
 8
 9

 1
 2
 3
 4
 5
 6
 7
 8
 9
 10
 11

 3
 4
 5
 6
 7
 8
 9
 10
 11

 3
 4
 5
 6
 7
 8
 9
 10
 11

 8
 9
 9
 10
 11
 10
 10
 11

#### EXERCISES

Rows

Copy and complete David's chart, then name the numbers.

- 1. Go along the 4 row on your chart until—you come to the 5 column. Name the number in that square.  $4 + 5 = \boxed{a}$
- 2. Go along the 6 row until you come to the 8 column. Solve the equation:  $6 + 8 = \overline{u}$
- 3. Use your chart to name these sums:
  - **a.** 8 + 4, **b.** 9 + 6, **c.** 3 + 8, **d.** 7 + 4,
  - e. 6 + 5, f. 5 + 8, g. 9 + 7, h. 4 + 3.
- **4.** Find the number 9 in the 4 row. Solve the equation:  $9 4 = \mathbf{r}$
- 5. Find the number 12 in the 5 row. Solve the equation: 12 5 = 5

#### Addends and sums



When you add, you begin with two addends and name a sum. When you subtract, you begin with a sum and one addend and name the other addend. We can write most addition-subtraction facts in four ways using the set of two addends and their sum. For example, using {2, 6, 8} we get

$$2 + 6 = 8$$
  $8 - 6 = 2$   $8 - 2 = 6$ 

#### EXERCISES

Write four equations for each of these sets of two addends and a sum.

Write two equations for each of these sets of two addends and a sum.

Look at the pattern, then answer the questions.

, more district the questions.									
1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20

What is:

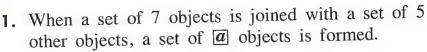
- **17.** 1 doubled?
- **18**. 2 doubled?
- **19.** 3 doubled?

- **20.** 6 doubled?
- **21.** 8 doubled?
- **22.** 9 doubled?

Numbers such as 2, 4, 6, 8, . . . are called even numbers. Can you state the general rule?

23. When both addends are the same, the sum is an ? number.

#### CHECKING UP



2. When a set of 8 objects is removed from a set of 12 objects, a set of e objects is left.

3. A pencil 7 inches long is x inches longer than a pencil 5 inches long.

**4.** The sum of 7 and 0 is **@**; of 9 and 0 is **@**; of 486 and 0 is **©**.

5. Other names for 7 include  $6 + \overline{a}$ ,  $5 + \overline{r}$ ,  $4 + \overline{c}$ .

**6.** Other names for 9 include  $8 + \overline{n}$ ,  $7 + \overline{e}$ ,  $6 + \overline{a}$ .

7. Other names for 11 include 10 + 5, 9 + 7, 8 + 2.

8. When you add, you begin with two \_\_? and name a \_\_? \_\_.

9. When you subtract, you begin with a sum and one \_\_?\_\_, and name a missing \_\_?\_\_.

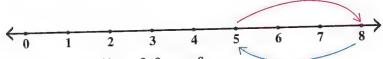
10. Name the rule, then copy and complete these number pairs.

a.  $\{(3, 8), (6, 11), (7, \mathbf{a}), (9, \mathbf{n}), (8, \mathbf{s}), (4, \mathbf{x})\}$ 

**b.**  $\{(1, 8), (3, 10), (2, \mathbf{e}), (4, \mathbf{o}), (6, \mathbf{u}), (7, \mathbf{a})\}$ 

**c.**  $\{(3, 9), (4, 10), (2, \overline{c}), (5, \overline{r}), (7, \overline{v}), (8, \overline{z})\}$ 

11. Write equations for the addition and subtraction shown on the number line.



12. Complete the family of facts for

**a.** 3 + 9 = 12

**b.** 10 - 6 = 4.

**13.** Since 7 < 10, 7 is to the \_\_\_? of 10 on the number line.

14. If you subtract 7 from 12 and then add 7 to your answer, your sum is  $\overline{z}$ .

### LOOKING BACK



### We have learned 45 addition-subtraction facts.

Name the sums.

$$1. 1 + 1$$

$$3.1 + 3$$

4. 
$$1 + 4$$

5. 
$$1 + 5$$

$$6.1 + 6$$

$$7. 1 + 7$$

$$8.1 + 8$$

10. 
$$4 + 4$$

11. 
$$4 + 5$$

12. 
$$4 + 6$$

13. 
$$4 + 7$$

15. 
$$4 + 9$$

16. 
$$2 + 2$$

$$17. 2 + 3$$

18. 
$$2 + 4$$

19. 
$$2 + 5$$

**20.** 
$$2 + 6$$

$$21. 2 + 7$$

**30.** 
$$3 + 9$$

$$31.5 + 5$$

**32.** 
$$5 + 6$$

33. 
$$5 + 7$$

$$34.5 + 8$$

**40.** 
$$7 + 7$$

Write the other three equations for each of these equations.

**46.** 
$$3 + 6 = 9$$

**47.** 
$$7 + 9 = 16$$

**48.** 
$$4 + 9 = 13$$

**49.** 
$$6 + 5 = 11$$

**50.** 
$$6 + 7 = 13$$

**51.** 
$$15 = 8 + 7$$

**52.** 
$$14 - 6 = 8$$

**53.** 
$$12 - 9 = 3$$

Every number has many names.

We know that 7 + 1 and 6 + 2 are other names for 8, so 7 + 2 and 6 + 3 are other names for  $\boxed{n}$ .

complete the names for the numbers on the left.

**55.** 12: 
$$8 + \mathbf{z}$$
,  $6 + \mathbf{a}$ ,  $9 + \mathbf{c}$ ,  $7 + \mathbf{e}$ 

**56.** 10: 
$$7 + 0$$
,  $m + 9$ ,  $12 - r$ ,  $14 - s$ 

### EXTRA FOR EXPERTS

Name the sums.

1. 
$$3 + 4 + 5 + 6$$

2. 
$$6+5+4+3$$

3. 
$$3 + 9 + 4 + 8$$

4. 
$$7 + 2 + 3 + 4$$

5. 
$$4 + 3 + 2 + 7$$

**6.** 
$$4 + 6 + 8 + 2$$

7. 
$$15 + 4 + 3 + 2$$

10. 
$$13 + 5 + 6 + 3$$

Name the missing addends.

13. 
$$3 + 4 + \mathbf{r} = 12$$

14. 
$$6+2+\overline{c}=17$$

15. 
$$3 + 6 + e = 18$$

**16.** 
$$6 + 1 + \overline{n} = 13$$

17. 
$$4+9+x=18$$

18. 
$$4 + 5 + \overline{r} = 16$$

19. 
$$s + 9 + 3 = 17$$

**20.** 
$$u + 3 + 3 = 17$$

**21.** 
$$v + 3 + 4 = 15$$

**22.** 
$$x + 5 + 3 = 15$$

**23.** 
$$\boxed{z} + 8 + 4 = 16$$

**24.** 
$$a + 6 + 2 = 17$$

Name the sums or missing addends.

- **32.** Here are three addends whose sum is 10: {3, 3, 4}. Name 8 other sets of three addends whose sum is 10.
- **33.** Here are three addends whose sum is 12: {3, 4, 5}. Name 9 other sets of three addends whose sum is 12.
- 34. The digit 7 appears in these basic addition facts: 3 + 4 = 7, 5 + 2 = 7, 6 + 1 = 7, 7 + 1 = 8, 7 + 2 = 9, 7 + 3 = 10, 7 + 4 = 11, 7 + 5 = 12, 7 + 6 = 13, 7 + 7 = 14, 7 + 8 = 15, 7 + 9 = 16, 8 + 9 = 17.

In how many basic addition facts does the digit 8 appear?

In how many basic subtraction facts does the digit 8 appear?

### EXTRA FOR EXPERTS



Solve the equations; then complete the patterns.

1. 
$$4 + 6 = a$$
 $3 + 9 = c$ 
 $2 + n = 0$ 
 $3 + 9 = c$ 
 $3 + 10 + c$ 
 $3 + 10 + c$ 
 $4 + 10 + c$ 

2. 
$$3 + 7 = a$$
 $6 + 8 = n$ 
 $\boxed{5 + r} = 0$ 
 $\boxed{2 \cdot 3 \cdot 7 \cdot x}$ 
 $\boxed{6 \cdot 8 \cdot z}$ 
 $\boxed{c \cdot e \cdot n}$ 

Complete the number patterns.

3.	6	9	a
	3	4	n
	x	$\boldsymbol{z}$	e

Just For Fun

- 11. Play this Addition-Subtraction Game with your neighbor. One of you says, "I'm thinking of two addends. They are 3 and 4." The other one says, "The sum is 7." If he is right, he takes a turn. He may say, "I'm thinking of a sum and an addend. The sum is 8, the addend is 3." Then you will say, "The other addend is 5."
- 12. Make a list of times when you may need to add or subtract. Your list may include some of these: making change, finding how many for lunch, losing some of your books.
- 13. Make up puzzles like these:
  - **a.** Which number gives a sum of 8 when added to 3?
  - **b.** Which number gives a sum of 7 when added to 7?

# CHAPTER 3 · Geometry

### Curves, lines, line segments

These are **curves**.

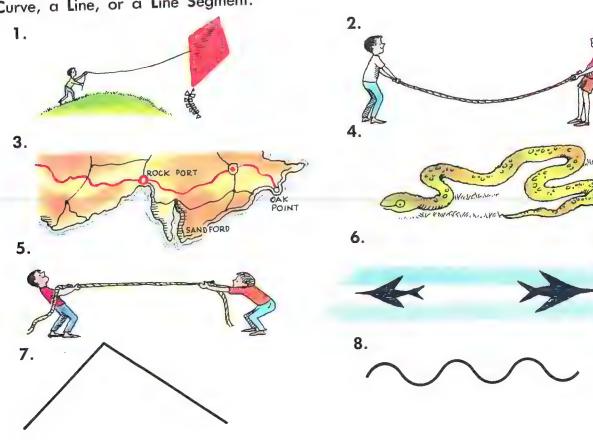
Some curves go on and on.

A **line** is straight and goes on and on.

A line segment is part of a line and has two ends.

#### EXERCISES

Write C, L or LS to tell whether the picture shows a Curve, a Line, or a Line Segment.



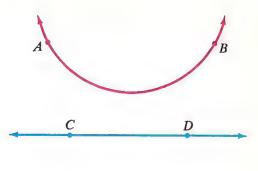
#### Points, lines, line segments

6

A and B name points on the curve. C and D name points on the line. The line may be named  $\overrightarrow{CD}$  or  $\overrightarrow{DC}$ . E and F name the end points on the line segment.

The line segment may be named  $\overline{EF}$  or  $\overline{FE}$ .

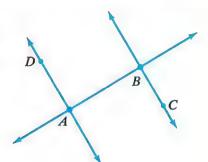
We must name two points to name a line or a line segment.



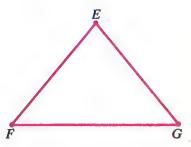
E F

#### EXERCISES

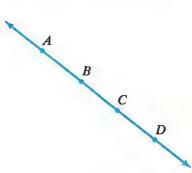
- 1. Name the point where  $\overrightarrow{AB}$  and  $\overrightarrow{BC}$  meet.
- **2.** Name the point where  $\overrightarrow{AB}$  and  $\overrightarrow{DA}$  meet.
- **3.** Is the point B on  $\overrightarrow{AD}$ ?
- **4.** Is the point C on  $\overrightarrow{AB}$ ?



- 5. Name the line segment with end points E and F.
- **6.** Name the line segment with end points F and G.
- **7.** Name the point where  $\overline{EF}$  and  $\overline{FG}$  meet.

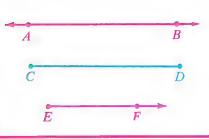


- 8. Two names for this line are  $\overrightarrow{AB}$  and  $\overrightarrow{AC}$ . Can you write three other names for this line?
- **9.**  $\overline{AB}$  and  $\overline{AC}$  are on the line. Can you name three other line segments on the line?



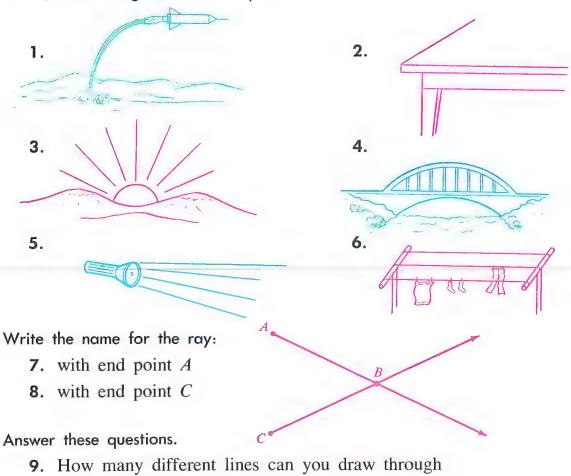
### End points

The line AB has no end points. The line segment CD has two end points.  $\overrightarrow{EF}$  is a ray. It has one end point, E.



#### EXERCISES

Write C, LS or R to tell whether the picture shows a Curve, a Line Segment, or a Ray.



10. How many rays can you draw with the same point

64 (sixty-four)

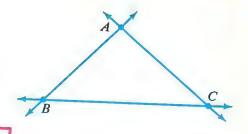
one point?

as their end point?

## Drawing lines, line segments, rays



The points A, B, and C show where the lines meet. The lines are  $\overrightarrow{AB}$ ,  $\overrightarrow{BC}$ , and  $\overrightarrow{CA}$ . The points A, B, and C are the end points of the line segments  $\overrightarrow{AB}$ ,  $\overrightarrow{BC}$ ,  $\overrightarrow{CA}$ .



#### EXERCISES

#### Draw two points.

- 1. Use your ruler to draw a line through them.
- 2. How many different lines can you draw which go through both of the points?
- **3.** How many different line segments can you draw with these points as end points?

## Draw three points and label them A, B, C.

- **4.** Use your ruler to draw lines through the points A and B, B and C, A and C.
- 5. How many lines did you draw?
- **6.** Can you draw any other lines through two of these points?

## Draw three points and label them D, E, F.

- **7.** Use your ruler to draw lines through the points *D* and *E*, *E* and *F*, *D* and *F*.
- **8.** How many different lines did you draw?

## Draw four points and label them $G,\ H,\ I,\ J.$

**9.** How many lines can you draw if each line must pass through two of these points?

B

D E F

 $\bullet G$ 

A

 $\bullet$ 

 $\bullet I$ 

J

·C

#### **Planes**



This **surface** is flat. It is part of a **plane**.



This surface is not flat. It is not part of a plane.

#### EXERCISES

Write the numerals 1-6. Next to each numeral write **P** or **NP** to tell whether the picture shows part of a plane or not.

1. 3.

4.

2.

\*\* H

5.

6.

Solve the problems.

7. Bob thinks he has a rule to decide whether a surface is part of a plane or not. He says that if he draws a line segment through any two points in the surface and the line segment lies entirely in the surface, the surface is a plane. Check Bob's rule with these two surfaces.

a. a ball

- b. the chalkboard
- 8. If we take any two points on the surface of the ball, can we join them with a line segment without going through the ball? Can a line segment lie in the surface of the ball?
- **9.** If we take any points on the chalkboard and join them with a line segment, will the line segment lie in the chalkboard?



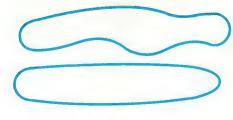
Look at these curves in a plane.











Curves

Closed curves

Simple closed curves

#### EXERCISES

Write the numerals 1-9. Next to each numeral write C or NC to tell whether the curve is Closed or Not Closed.

1.



2.



3.



4.



5.



6.



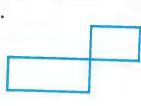
7.



8.



9.



#### Answer the questions.

- 10. Name the closed curves above which are simple closed curves.
- 11. Draw a curve which is not closed.
- 12. Draw a simple closed curve.
- **13.** Draw a closed curve which is not a simple closed curve.

1		

To measure the length of  $\overline{AB}$ , we choose a **unit of** length and match it with  $\overline{AB}$ . We ask "How many units match  $\overline{AB}$ ? If 4 units match  $\overline{AB}$ , we write  $\overline{m(AB)} = 4$ . The m stands for the **measure** of  $\overline{AB}$  and means "the number of units of length in  $\overline{AB}$ ."

	EXERCISES	AD and means
		How many in:
	2	1
	4	3
	6	5
ece	unit, measure these lengths.	Mark off the unit length of cardboard. Using this

7.  $m(\overline{AB}) = \overline{a}$ 

8.  $m(\overline{CD}) = \overline{C}$ 

9.  $m(\overline{EF}) = e$ 

 $10. \ \ m(\overline{GH}) = \boxed{n}$ 

11.  $m(\overline{IJ}) = \overline{v}$ 

Answer the question.

12. Wendy and Bob measured the classroom. Using his paces Bob said, "The measure is 12."
Using her paces Wendy said, "The measure is 14." Why were their measures different?



1 2 3 4 5

The unit is an inch.  $m(\overline{AB}) = 5$ . The ruler is 5 inches long.

#### EXERCISES

Name the length of each object in inches.

- 1.
- 2.
- 3.
- 5.



Use your ruler to draw line segments:

- 6. 1 inch long
- 7. 2 inches long
- 8. 7 inches long

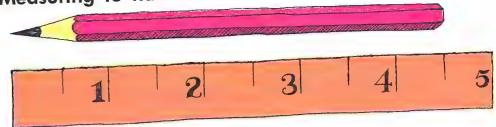
- 9. 3 inches long12. 5 inches long
- 10. 4 inches long13. 6 inches long
- 11. 8 inches long14. 9 inches long

Jse your ruler and a pair of scissors to cut strips of ardboard:

- 15. 1 inch long
- 16. 2 inches long
- 17. 7 inches long

- 18. 3 inches long21. 5 inches long
- 19. 4 inches long22. 6 inches long
- 20. 8 inches long23. 9 inches long
- 24. Compare your strips with the line segments you drew. Are they the same length?





The marks between the inches are for the **half inches**. The measure of the pencil is  $4\frac{1}{2}$ . This is read "Four and one half." The length of the pencil is  $4\frac{1}{2}$  inches.

#### EXERCISES

Use your ruler to measure the lengths of these objects.

1.

2. Emmannami

3.

Draw line segments:

- **4.**  $7\frac{1}{2}$  inches long
- 5.  $4\frac{1}{2}$  inches long
- **6.**  $3\frac{1}{2}$  inches long

- 7.  $5\frac{1}{2}$  inches long
- **8.**  $1\frac{1}{2}$  inches long
- 9.  $8\frac{1}{2}$  inches long

In many countries another unit of length is used. It is a little smaller than half an inch. It is called a **centimeter**.

entimeter.

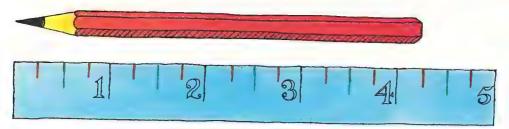
A B C D

 $\overline{AB}$  is 1 centimeter long,  $\overline{AC}$  is 4 centimeters long,  $\overline{AD}$  is 10 centimeters long. Cut out a strip of cardboard and use the line segments to make a ruler to measure up to 20 centimeters.

Use a centimeter ruler to measure these line segments.

10. \_\_\_\_\_ 11. \_\_\_\_

## Measuring to one fourth of an inch



The marks between the inches are for the half inches and fourth inches. The half inch marks are in blue; the fourth inch marks are in red. If the unit is inches, the measure of the pencil is  $4\frac{1}{4}$ . This is read "four and one fourth inches".

#### EXERCISES

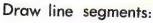
Use your ruler to measure the length of these objects to a fourth of an inch.



2.



3.

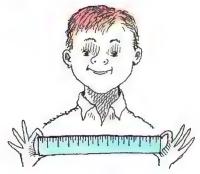


- **4.**  $1\frac{1}{4}$  inches long **5.**  $2\frac{1}{4}$  inches long
  - **6.**  $3\frac{1}{4}$  inches long
- 7.  $5\frac{1}{4}$  inches long 8.  $7\frac{1}{4}$  inches long
- 9.  $4\frac{3}{4}$  inches long

Name these lengths.

- 10. Measure the length of four objects in your classroom.
- 11. Measure the box, whistle and crayon again, using a centimeter as your unit. State the measures as whole numbers of centimeters.

#### Measuring



Do you think Bill should use inches as his unit for measuring the width of the room? Why not? Do you know a larger unit he could use?

Some rulers form a larger unit. Does your ruler show 12 inches? If so, it forms a unit called a **foot**. Is your 12 inches long? Do you know anyone whose is 12 inches long?

#### EXERCISES

1. Use a foot rule to name the length and width of your classroom. Make a list of other objects and measure them with your foot rule. Here are some: your arm from the wrist to the elbow, your leg, the leg of a table, the width of a door, the height of a desk.

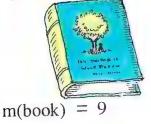
Write inches or feet to show the units in which these objects were measured.

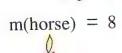




$$m(car) = 16$$

4.

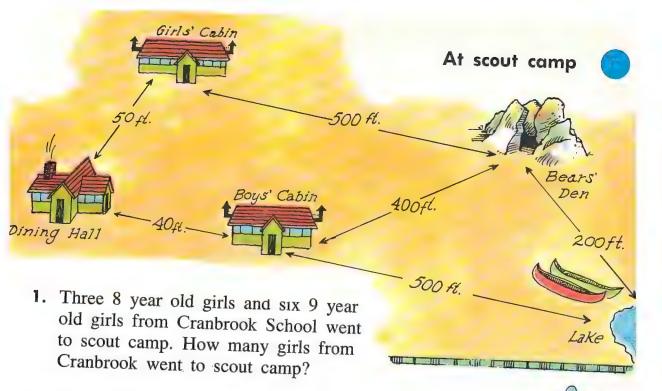




5.



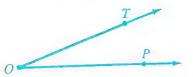
72 (seventy-two)



- 2. The total number of boys and girls that went to scout camp from Cranbrook was 17. How many boys went?
- 3. Study the map above. Do the boys or girls have the greater distance to walk to the Dining Hall?
- 4. Using the shortest path, how far do the boys have to walk to the Bears' Den?
- 5. Which path is longer:
  - a. Dining Hall to Lake or Bears' Den to Lake?
  - **b.** Bears' Den to Boys' Cabin or Girls' Cabin to Boys' Cabin?
  - c. Girls' Cabin to Lake or Boys' Cabin to Bears' Den?
- 6. Which way is better for the girls to walk to get to the Bears' Den from their cabin: a. Girls' Cabin to Dining Hall to Boys' Cabin to Bears' Den? or b. Girls' Cabin to Bears' Den?

#### Angles

This is an **angle**. It is made up of **two rays** with a common end point called the **corner point**.

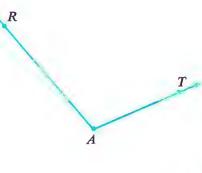


This angle is made up of the two rays  $\overrightarrow{OT}$  and  $\overrightarrow{OP}$ . We may name this angle as  $\angle TOP$  or as  $\angle POT$ .

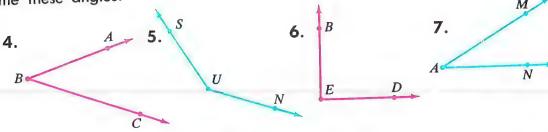
#### EXERCISES

Study the angle on the right.

- 1. Name the two rays which make up the angle.
- 2. Do these rays have the same end point?
- **3.** One name for this angle is  $\angle TAR$ . What is the other name for this angle?



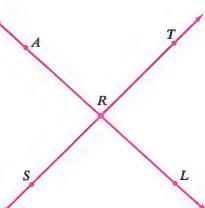
Name these angles.



**8 -11.** Name the pair of rays for each angle in exercises 4-7.

Study the figure on the right.

- 12.  $\overrightarrow{AL}$  and  $\overrightarrow{ST}$  are lines. They meet at the point  $\overline{a}$ .
- **13.**  $\overrightarrow{RA}$  is a ray. Can you name 3 other rays from R?
- 14.  $\angle TRL$  is an angle. Can you name 3 other angles in this figure?



#### Right angles

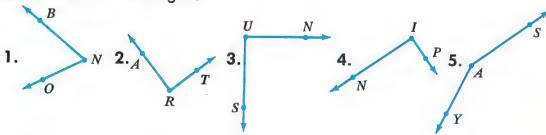




These are special angles. They are called **right** angles. Can you find examples of right angles in your classroom?

#### EXERCISES

Name each of these angles.



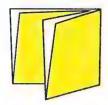
**6.** Which of the angles in 1-5 are right angles?

Make a right angle.

**Step 1.** Take a sheet of paper. Fold it like this.



**Step 2.** Fold the paper again so the first folds meet, like this.



- Step 3. Unfold the paper.
- 7. The four angles in the middle of your paper are \_\_? angles.

#### Parallel lines

Here are two lines. Do they meet?



Two lines like  $\overrightarrow{AB}$  and  $\overrightarrow{CD}$  that do not meet are called parallel lines. When two lines are parallel, any line segments in them are also parallel, like  $\overline{AB}$  and  $\overline{CD}$  in our diagram.

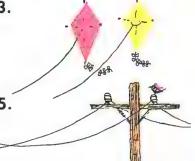
#### EXERCISES

Name the pictures which show parallel lines or parallel line segments.

2.

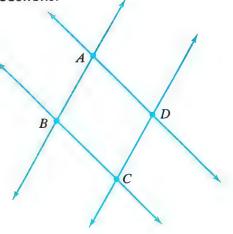


3.



Study the figure on the right, then answer the questions.

- **6.** AB is parallel to \_\_?\_.
- 7.  $\overrightarrow{AD}$  is parallel to  $\underline{?}$ .
- **8.**  $\overrightarrow{DA}$  and  $\overrightarrow{BC}$  are called  $\underline{\phantom{A}}$ ?
- **9.**  $\overrightarrow{DA}$  and  $\overrightarrow{DC}$  form an angle. Name this angle.
- 10.  $\overline{DC}$  is a line segment. Name a line segment parallel to  $D\overline{C}$ .
- 11.  $\overline{BC}$  is a line segment. Name a line segment parallel to  $B\overline{C}$ .

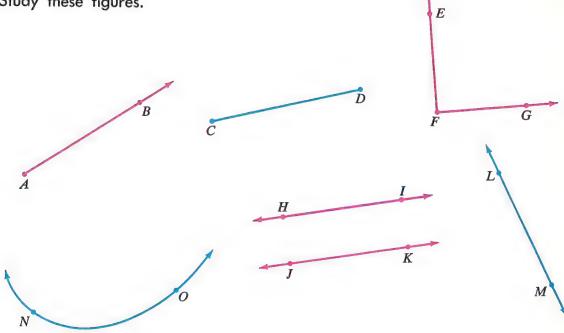


4.

#### Practice page



Study these figures.



- 1. Name the line.
- 3. Name the ray.
- 5. Name the curve that is not part of a line.
- 2. Name the right angle.
- 4. Name the line segment.
- 6. Name the pair of parallel lines.
- 7. A line has \_\_? end points. A ray has \_\_? end points.
- **8.** A line segment has \_\_? end points.

Use your ruler to measure these line segments in inches.

9. 
$$m(\overline{ST}) = \overline{a}$$

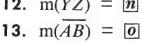
10.  $m(\overline{UV}) = \overline{c}$ 



11. 
$$m(\overline{WX}) = e$$



12. 
$$m(\overline{YZ}) = \overline{n}$$

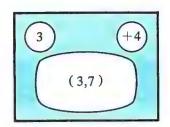




#### On the screen, a number pair

The third grade at Brookpark School has a special TV set. One knob tunes in a **number**. The other knob tunes in a **rule**. The screen shows a **number pair**.

Betty tells Joe how the TV set works. She says, "The number knob gives the first number in the number pair. The rule "+4" means add 4 to the first number in the pair. 3 + 4 = 7, so, if 3 is the first number, 7 is the second number. The number pair is (3, 7)."



#### EXERCISES

Use the TV set to complete each number pair.

4 (4,a) (4,a)

(6, a)

(8,<u>a</u>)

(5,a)

6 (43) (6,a) (7,a)

(9,a)

(5,a) (5,a)

Complete these number pairs using the rule "+ 5."

- **9.** (2, **a**)
- **10.** (3, **r**)
- **11.** (4, **C**)
- **12.** (5, **S**)

- **13**. (6, **e**)
- **14.** (7, **n**)
- 15.  $(0, \mathbf{u})$
- 16. (8, **v**)

Complete these number pairs.

- **17.**  $\{(2,5), (3,6), (6, \mathbf{a}), (7, \mathbf{n}), (8, \mathbf{c}), (9, \mathbf{0}), (4, \mathbf{e})\}$
- **18.**  $\{(2,8),(3,9),(6,\overline{r}),(7,\overline{s}),(8,\overline{u}),(9,\overline{v}),(5,\overline{z})\}$
- **19.**  $\{(2,9), (3,10), (6, \mathbf{x}), (7, \mathbf{c}), (8, \mathbf{a}), (9, \mathbf{n}), (5, \mathbf{o})\}$

78 (seventy-eight)

#### Practice page

Write in the short form.

1. 
$$6000 + 400 + 80 + 6$$
 2.  $5000 + 400 + 50$  3.  $4000 + 600 + 8$ 

**2.** 
$$5000 + 400 + 50$$

3. 
$$4000 + 600 + 8$$

Name the tens' digit in:

Name the thousands' digit in:

Complete the equations.

12. 
$$3 + 16 = 16 + \boxed{a}$$

**14.** 
$$800 + 40 = 40 + 6$$

**15.** 
$$.7 + 9 = 8 + \boxed{r}$$
 **16.**  $6 + 5 = 4 + \boxed{v}$ 

16. 
$$6 + 5 = 4 + [v]$$

17. 
$$8 + 3 = n + 4$$

18. 
$$r + 6 = 10 + 4$$
 19.  $r + 9 = 8 + 6$  20.  $8 + s = 9 + 4$ 

19. 
$$r + 9 = 8 + 6$$

**20.** 
$$8 + 5 = 9 + 4$$

21. Write eight different names for 8.

Complete the number sentences. Write <, >, or = to replace each O.

**22.** 
$$4+9\bigcirc 6+6$$
 **23.**  $7+8\bigcirc 16$ 

**24.** 
$$8 + 8 \bigcirc 6 + 9$$

**25.** 
$$9+3 \bigcirc 3+9$$
 **26.**  $8+7 \bigcirc 6+9$  **27.**  $9+5 \bigcirc 8+6$ 

**26.** 
$$8 + 7 \bigcirc 6 + 9$$

**27.** 
$$9 + 5 \bigcirc 8 + 6$$

**28.** 
$$8 + 3 \bigcirc 12$$

**28.** 
$$8 + 3 \bigcirc 12$$
 **29.**  $3 + 9 \bigcirc 6 + 6$  **30.**  $3 + 7 \bigcirc 6$ 

Name the missing addends.

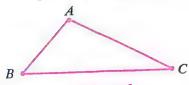
31. 18 32. 17 33. 16 34 
$$\frac{-9}{z}$$
  $\frac{-u}{7}$ 

16 **34.** 13 **35.** 14 **36.** 12 **37.** 14 
$$\frac{-\overline{w}}{7}$$
  $\frac{-\overline{m}}{7}$   $\frac{-8}{7}$ 

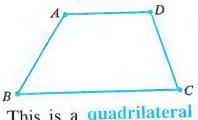
solve the problems.

- 38. Red picked 12 pieces of fruit, 8 apples and the rest pears. How many pears did he pick?
- 39. Fred picked 8 pears and 4 apples. Mary picked 6 pears and 9 peaches. Who picked more fruit?
- 40. Donna picked 7 apples and 7 peaches. Lucy picked 6 peaches and 8 apples. Did they pick the same number of pieces of fruit?

#### Joining line segments



This is a triangle.



This is a quadrilateral

The line segments that make up a triangle or quadrilateral are called sides. The end points of the sides are called corner points.

#### EXERCISES

Answer the questions.

- 1. How many sides does a triangle have?
- 2. How many corner points does a triangle have?
- 3. How many angles does a triangle have?
- 4. How many sides does a quadrilateral have?
- 5. How many corner points does a quadrilateral have?
- 6. How many angles does a quadrilateral have?

#### Just For Fun

1. Draw a triangle on a piece of paper using a pencil and ruler. Cut along the sides. Tear the piece of paper you have cut out into three parts, each with a corner. Fit the corners together. Do your pieces fit on a line segment-like ours?

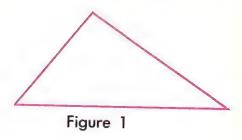


2. Draw a quadrilateral on a piece of paper. Cut along the sides. Tear the piece of paper you have cut out into four parts, each with a corner. Fit the corners together on a flat surface. Do your pieces fit like ours?



#### Making models

- 1. Cut 3 strips of cardboard of lengths 5 inches, 7 inches, and 9 inches. Make a triangle, like that in figure 1, by fastening the strips at the corners with paper fasteners. Can you change the shape of the triangle?
- 2. Cut 3 strips of cardboard of lengths 4 inches, 5 inches, and 10 inches. Can you make a triangle with these strips? Why not?
- 3. Cut 4 strips of cardboard of lengths 4 inches, 5 inches, 7 inches and 8 inches. Make a quadrilateral with paper fasteners, like the one in figure 2. Can you change the shape of the quadrilateral?
- 4. Cut another strip of length 10 inches and use it to join opposite corner points as in figure 3. a. Can you change the shape of this quadrilateral? b. How many triangles can you see in your new quadrilateral?
- 5. Cut 2 strips of cardboard 5 inches long and 2 strips of cardboard 7 inches long. Join the strips with paper fasteners so that the opposite sides are of the same length, as in figure 4.
  - **a.** Can you change the shape of this quadrilateral? **b.** Do you think the opposite sides are parallel?
- bridge? If not, why not? b. What would a builder have to do to make it a good bridge?



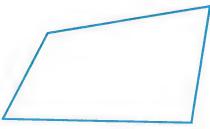


Figure 2



Figure 3

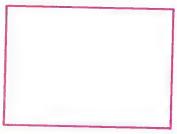
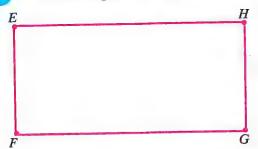


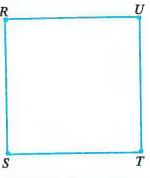
Figure 4

#### Rectangles, squares



This is a rectangle.

It is a special quadrilateral. What do you notice about the angles?



This is a square.

It is a special rectangle. Wha do you notice about the sides

#### EXERCISES

Study rectangle ABCD, then answer the questions.

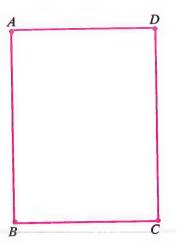
1. 
$$m(\overline{AB}) = \overline{C}$$
 2.  $m(\overline{DC}) = \overline{e}$ 

2. 
$$m(\overline{DC}) = \overline{e}$$

3. 
$$m(\overline{AD}) = \overline{n}$$

4. 
$$m(\overline{CB}) = \overline{x}$$

- **5.**  $\overline{AB}$  and  $\overline{DC}$  are parallel. Two other line segments which are parallel are [r] and [s].
- **6.**  $\angle ADC$ ,  $\angle CBA$ , and  $\angle BAD$  are all ? angles.



Study square EFGH, then answer the questions.

7. 
$$m(\overline{EF}) = \overline{n}$$

8. 
$$m(\overline{E}\overline{H}) = \overline{x}$$

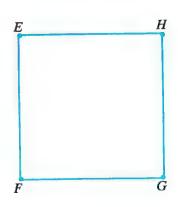
9. 
$$m(\overline{GH}) = \overline{r}$$

10. 
$$m(\overline{FG}) = S$$

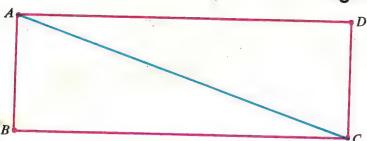
11. 
$$\overline{EF}$$
 is parallel to  $\overline{u}$ .

**12.** 
$$\overline{EH}$$
 is parallel to  $\overline{v}$ .

13. 
$$\angle EHG$$
 is a right angle. Three other right angles are  $\boxed{a}$ ,  $\boxed{c}$  and  $\boxed{e}$ .



Diagonal lines



This is a rectangle. The line segment AC is called a diagonal of the rectangle. How many triangles do you see? Does this rectangle have another diagonal?

#### EXERCISES

Name the measures for rectangle POLK.

$$1. \ \mathrm{m}(\overline{PO}) = \overline{a}$$

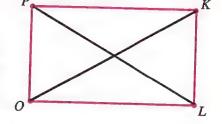
1. 
$$m(\overline{PO}) = \overline{a}$$
 2.  $m(\overline{KL}) = \overline{c}$ 

3. 
$$m(\overline{PK}) = e$$
 4.  $m(\overline{OL}) = n$ 

4. 
$$m(\overline{OL}) = \overline{n}$$

$$5. \ \mathrm{m}(\overline{PL}) = \overline{r}$$

5. 
$$m(\overline{PL}) = \overline{r}$$
 6.  $m(\overline{OK}) = \overline{s}$ 



Name the measures for square DART.

7. 
$$m(\overline{DA}) = x$$

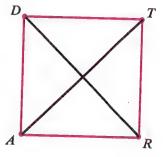
8. 
$$m(\overline{AR}) = [\overline{u}]$$

$$9. \ \mathrm{m}(\overline{RT}) = \overline{v}$$

9. 
$$m(\overline{RT}) = \overline{v}$$
 10.  $m(\overline{TD}) = \overline{z}$ 

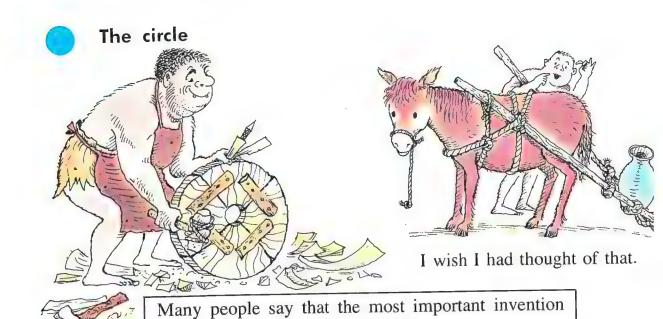
11. 
$$m(\overline{DR}) = \overline{a}$$

11. 
$$m(\overline{DR}) = \overline{a}$$
 12.  $m(\overline{AT}) = \overline{s}$ 

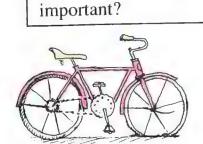


Complete these sentences.

- 13. The opposite sides of a rectangle have the same \_\_\_? .
- 14. The diagonals of a rectangle have the same
- 15. If a rectangle has four sides of the same measure it is called a \_ ? .



ever made was the wheel. Why was it so

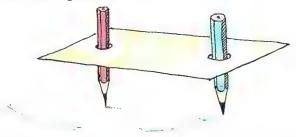


A wheel is not of much use unless it is **round**.

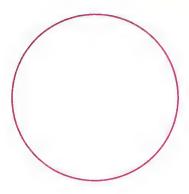


A wheel is not of much use unless it has a **center**.

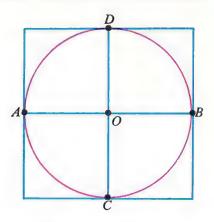
A circle is perfectly round. Every point on the circle is the same distance from the center point. You can draw a circle by making two holes in a piece of cardboard. Put a pencil in each hole. Hold one pencil still. (That point will be the center.) Swing the other pencil around like this.



#### The diameter and radius of a circle



This is a circle

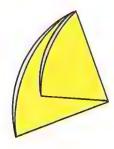


It fits inside this square

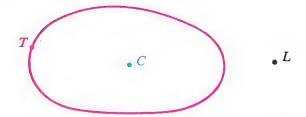
 $\overline{AB}$  and  $\overline{CD}$  join the points where the circle touches the square. The measure of these line segments is the measure of the distance across the circle.  $\overline{AB}$  and  $\overline{CD}$  are called **diameters**. A diameter passes through the **center** of the circle. Each of the line segments,  $\overline{OA}$ ,  $\overline{OB}$ ,  $\overline{OC}$  and  $\overline{OD}$  is called a **radius**.

#### EXERCISES

- Use a cardboard strip to draw a circle with a radius of:
  - a. 1 inch
  - **b.** 2 inches
  - c. 3 inches
  - d. 4 inches
- Praw a circle on a piece of paper. Cut around the circle so that you have a paper disc. Fold the disc twice as shown. What do you know about the point where the folds meet?



#### Inside-outside



T is a point on the curve. C is a point inside the curve. L is a point outside the curve.

#### EXERCISES

Study figure 1, then name the points which are:

- 1. inside the curve
- 2. on the curve
- 3. outside the curve

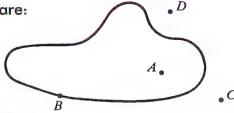
Study figure 2, then name the points which are:

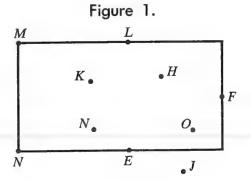


- 5. inside the curve
- 6. outside the curve

Copy figure 3, then answer the questions.

- **7.** Join A and B. Is all of  $\overline{AB}$  inside the curve?
- **8.** Join A and D. Is all of  $\overline{AD}$  inside the curve?
- **9.** Points E and F are on the curve. Is all of  $\overline{EF}$  on the curve?
- **10.** Join F and G. Is all of  $\overline{FG}$  on the curve?
- 11. Join D and C. Is all of  $\overline{DC}$  outside the curve?
- **12.** Can you join A and C without going through the curve?





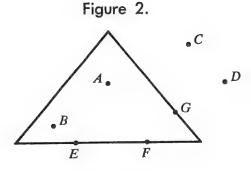
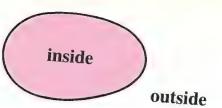


Figure 3.



Regions

The part of a plane inside a simple closed curve is called a region.









Rectangular Region

Square Region

#### EXERCISES

Write the numerals 1-6. Next to each numeral, write circular, triangular, rectangular, or square, to describe each region.

1.

3.

6.

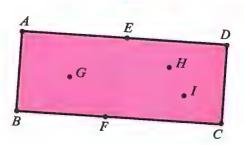
2.

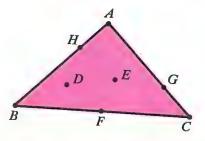




tudy the figures on the right.

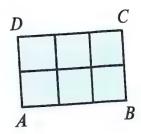
- 7. Name the points on the rectangle.
- 8. Name the points in the rectangular region.
- 9. Name the corner points of the rectangle.
- 10. Name the points on the triangle.
- 1. Name the points in the triangular region.
- 2. Name the corner points of the triangle.





## Measuring regions

We measure the **area** of a **region**. The unit of area is a segment of the plane. If our unit of area is  $\square$ , we ask, "How many of these square regions are there in the rectangular region? m(ABCD) = 6



## EXERCISES

Using the small squares as the unit of area, name the measures of these regions.

1.



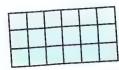
2.



3.



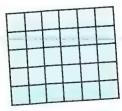
4.



5.



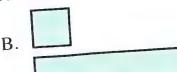
6



7. Should you use A, B, or C to measure the region?



Α.



C. L



The small square is the unit of area. By counting, name the measure of these regions.



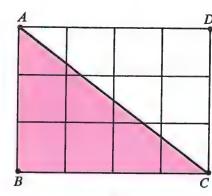
$$2. m(ABC) = \boxed{c}$$

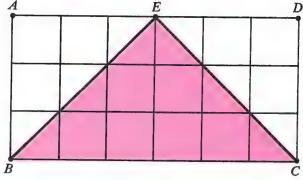
$$3. m(ABCD) = n$$

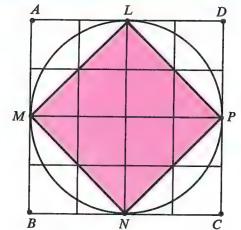
4. 
$$m(BCE) = \mathbf{r}$$



**6.** 
$$m(LMNP) = s$$







**7.** What can you say about the area of the circle compared with the area of *ABCD* and the area of *LMNP*?

#### Practice page

Write four equations for each of these sets of two addends and a sum.

Complete the names for the number on the left.

**13.** 12: 
$$4 + \overline{a}$$
,  $7 + \overline{c}$ ,  $6 + \overline{e}$ ,  $9 + \overline{n}$ 

14. 8: 
$$5 + \overline{a}$$
,  $3 + \overline{c}$ ,  $2 + \overline{e}$ ,  $8 + \overline{n}$ 

Name the missing addends.

15. 12 16. 14 17. 13 18. 7 19. 9 20. 15 21. 7 
$$\frac{-6}{a}$$
  $\frac{-8}{c}$   $\frac{-e}{5}$   $\frac{+n}{16}$   $\frac{+u}{12}$   $\frac{-7}{r}$   $\frac{+z}{11}$ 

Complete the number sentences. Write <, >, or = to replace each O.

**22.** 
$$7+6 \bigcirc 8+7$$

**24.** 
$$8 + 6 \bigcirc 7 + 7$$

**22.** 
$$7+6 \ 8+7$$
 **23.**  $3+9 \ 2+9$  **24.**  $8+6 \ 7+7$  **25.**  $7-3 \ 12-8$  **26.**  $4+2 \ 13-6$  **27.**  $5+8 \ 2+8$ 

Name the sums or missing addends.

Solve the problems.

- 35. Jonny and Ricky had a race to see who could pick more fruit in a minute. Jonny picked 3 apples, 4 peaches, and 6 pears. Ricky picked 6 apples, 2 peaches and 8 pears. Who won the race?
- **36.** There are 6 pear trees, 11 apple trees and 9 peach trees. How many more apple trees are there than pear trees? How many fewer peach trees are there than apple trees?

#### CHECKING UP



Write the numerals 1-4. Next to each numeral write C or NC to show which of these curves are Closed and which are Not Closed.

1. /	1

2.



3.





Measure the lengths of these line segments.

Answer the question.

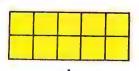
9. Barry said "I have drawn a line." Bobby said that Barry couldn't have drawn a line. Was Bobby right?

Write the numerals 10-17. Next to each numeral vrite a letter from a-h, to match the names on the eft with the figures on the right.

- 10. rectangle
- 11. closed curve
- 12. line segment
- 13. parallel lines
- 14. angle
- 15. the diameter of a circle
- 16. a ray
- 17. measuring the area









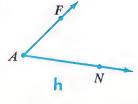
g

D



 $\boldsymbol{E}$ 





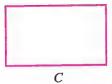
(ninety-one) 91

#### LOOKING BACK

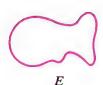












Each of these is a closed curve.

The figure labeled:

1. *A* is called a \_\_?\_\_

**3.** *C* is called a \_\_\_?\_\_

**2.** *B* is called a \_\_?\_\_

**4.** *D* is called a \_\_?

There are two parts to any measurement, a unit of measurement and a number called the measure.

5. Measure the line segments in inches.

$$m(\overline{AB}) = \overline{a} \stackrel{A}{\longleftarrow}$$

$$m(\overline{CD}) = C$$

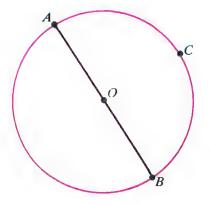
D

6. Measure the region in square units.

$$m(ABCD) = e^{A}$$

A circle is a set of points each of which is the same distance from a point called the center.

- **7.** Which point is the center of the circle?
- **8.** Is the distance from C to O the same as the distance from A to O? from B to O?
- 9.  $\overline{OA}$  is called a  $\underline{?}$  of the circle.
- **10.**  $\overline{AB}$  is called a  $\underline{\phantom{AB}}$  of the circle.



## Paper folding and cutting

- 1. Take a rectangular piece of paper and fold each side the same distance from the edges, as in our diagram. Cut along the dotted line segments. Fasten the corners with glue or paper clips to make an open box.
- **2.** a. Fold a rectangular piece of paper along the dotted  $\overline{AC}$  in figure 1 so that B lies on D.
  - **b.** Cut from *D* to *C*, leaving a folded square.
  - **c.** Fold so that A lies on C as in figure 2.
  - **d.** Fold so that C falls on D.

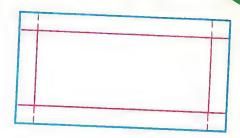
Can you see how to make a cup from there?

3. Take a rectangular strip of paper 8 times as long as it is wide. Fold it to form 8 squares as in figure 3. Then fold across the dotted line segments as in figure 4. Can you see how to make a closed box from there?

## Answer the question.

4. Once there was a very famous man who had two cats, a big one and a little one. He grew tired of opening his door to let the cats in and out. In his door he cut two holes, a big one and a little one. Do you think he was a good mathematician? Why not?





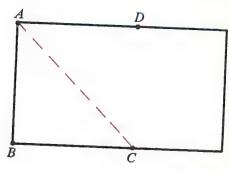
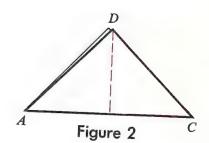
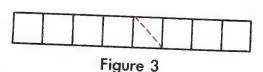
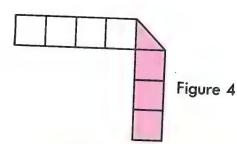


Figure 1







# 4 · Addition-Subtraction

# Addition, one addend less than ten

Study this addition.

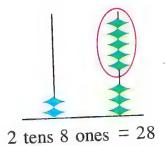
$$23 + 5 = (20 + 3) + 5$$

$$= 20 + (3 + 5)$$

$$= 20 + 8$$

$$= 28$$

$$\begin{array}{r}
 20 + 3 \\
 + 5 \\
 \hline
 20 + 8 = 28
 \end{array}$$

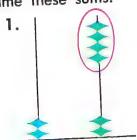


#### EXERCISES

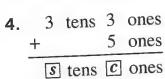
e tens z ones

**5.** 6 tens 4 ones

Name these sums.



a tens c ones



v tens u ones 70 + 7

8.

2.

2 ones

3.

 $\overline{c}$  tens  $\overline{n}$  ones

Copy and complete these equations.

11. 
$$32 + 6 = (30 + \boxed{s}) + 6$$
  
=  $30 + (\boxed{s} + 6)$   
=  $30 + \boxed{v}$   
=  $\boxed{x}$ 

Solve these equations.

**13.** 
$$3 + 5 = 8$$
, so  $43 + 5 = \boxed{a}$ 

15. 
$$4 + 5 = 9$$
, so  $44 + 5 = \boxed{r}$ 

17. 
$$3 + 6 = 9$$
, so  $63 + 6 = \boxed{c}$ 

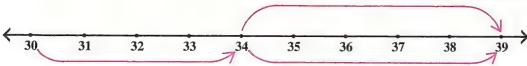
12. 
$$42 + 7 = (40 + \boxed{r}) + 7$$
  
=  $40 + (\boxed{r} + 7)$   
=  $40 + \boxed{n}$   
=  $\boxed{z}$ 

14. 
$$1 + 8 = 9$$
, so  $71 + 8 = 5$ 

**16.** 
$$2 + 6 = 8$$
, so  $12 + 6 = \boxed{e}$ 

**18.** 
$$4 + 4 = 8$$
, so  $34 + 4 = \overline{u}$ 

#### **Addition**



On the number line we see that 34 + 5 = 30 + (4 + 5)= 30 + 9= 39

#### EXERCISES

Use the number line above to name the sums and missing addends.

1. 
$$31 + 4 = a$$

**2.** 
$$31 + 6 = C$$

3. 
$$31 + 7 = [e]$$

4. 
$$31 + 8 = e$$

5. 
$$31 + \mathbf{u} = 33$$

**6.** 
$$37 + \boxed{a} = 39$$

7. 
$$31 + \overline{v} = 35$$

8. 
$$31 + \boxed{x} = 38$$

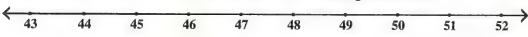
9. 
$$31 + \boxed{z} = 31$$

**10.** 
$$38 + \mathbf{c} = 39$$

11. 
$$34 + e = 36$$

12. 
$$33 + \overline{n} = 37$$

Use the number line to name the sums and missing addends.



13. 
$$47 + 2 = 0$$

14. 
$$48 + 1 = |r|$$

15. 
$$43 + 5 = \boxed{s}$$

16. 
$$43 + 6 = [n]$$

17. 
$$43 + \overline{v} = 43$$

18. 
$$44 + x = 47$$

19. 
$$43 + \boxed{a} = 47$$

**20.** 
$$43 + \mathbf{c} = 45$$

**21.** 
$$43 + [e] = 49$$

**22.** 
$$44 + r = 48$$

**23.** 
$$45 + \boxed{s} = 46$$

**24.** 
$$43 + \overline{u} = 44$$

Copy and complete this number pattern.

							•
25.	+	1	2	3	4	5	6
	21	22	23	24	a	C	e
	42	43	44	n	x	r	S
	53	54	55	u	v	0	z

Name the rule, then complete these number pairs.

**27.** 
$$\{(1, 6), (11, 16), (21, \mathbf{x}), (41, \mathbf{r}), (51, \mathbf{s}), (71, \mathbf{u})\}$$

**28.** 
$$\{(3, 7), (23, 27), (33, \overline{w}), (53, \overline{x}), (63, \overline{z}), (83, \overline{a})\}$$

**29.** 
$$\{(4, 9), (24, 29), (54,  $\overline{a}), (64, \overline{c}), (74, \overline{n}), (94, \overline{x})\}$$$

#### Subtracting a number less than ten

Study this subtraction.

$$38 - 5 = (30 + 8) - 5$$

$$= 30 + (8 - 5)$$

$$= 30 + 3$$

$$= 33$$

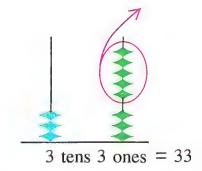
$$30 + 8$$

$$- 5$$

$$30 + 8$$

$$- 5$$

$$30 + 3 = 33$$



#### EXERCISES

Copy and complete these equations.

1. 
$$26 - 4 = (20 + 6) - 4$$
  
=  $20 + (6 - 4)$   
=  $20 + \boxed{a}$   
=  $\boxed{e}$ 

3. 
$$64 - 2 = (60 + \overline{n}) - 2$$
  
=  $60 + (\overline{n} - 2)$   
=  $60 + \overline{r}$   
=  $\overline{s}$ 

2. 
$$58 - 6 = (50 + 8) - 6$$
  
=  $50 + (8 - 6)$   
=  $50 + \boxed{c}$   
=  $\boxed{z}$ 

**4.** 
$$99 - 6 = (90 + \overline{u}) - 6$$
  
=  $90 + (\overline{u} - 6)$   
=  $90 + \overline{x}$   
=  $\overline{z}$ 

Name the missing addends.

7. 8 tens 9 ones
$$\frac{- \qquad 5 \text{ ones}}{\boxed{a \text{ tens } \boxed{c} \text{ ones}}} = \boxed{e}$$

8. 4 tens 3 ones
$$\frac{2 \text{ ones}}{u \text{ tens } x \text{ ones}} = z$$

Solve these equations.

**9.** 
$$7-2=5$$
, so  $17-2=\bar{a}$ ,  $37-2=\bar{o}$ ,  $97-2=\bar{c}$ 

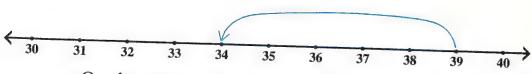
**10.** 
$$8 - 3 = 5$$
, so  $18 - 3 = \overline{m}$ ,  $48 - 3 = \overline{e}$ ,  $68 - 3 = \overline{x}$ 

11. 
$$9 - 2 = 7$$
, so  $29 - 2 = 5$ ,  $59 - 2 = 7$ ,  $79 - 2 = 7$ 

**12.** 
$$6-1=5$$
, so  $16-1=\overline{v}$ ,  $36-1=\overline{z}$ ,  $66-1=\overline{a}$ 

**13.** 
$$7-6=1$$
, so  $27-6=2$ ,  $87-6=1$ ,  $97-6=0$ 

#### Subtraction



On the number line we see that 39 - 5 = 34

#### EXERCISES

Use the number line above to name the missing addends.

1. 
$$36 - 2 = a$$

**2.** 
$$34 - 3 = c$$

3. 
$$37 - 2 = n$$

4. 
$$38 - 5 = 0$$

5. 
$$39 - 4 = \mathbf{r}$$

**6.** 
$$36 - 4 = \mathbf{u}$$

7. 
$$35 - \overline{v} = 31$$
  
10.  $37 - \overline{c} = 34$ 

8. 
$$36 - x = 34$$
11.  $33 - e = 32$ 

9. 
$$39 - \boxed{z} = 32$$
12.  $34 - \boxed{n} = 32$ 

Use the number line to name the missing addends.



**13.** 
$$54 - 3 = a$$
 **14.**  $57 - 2 = c$ 

14. 
$$57 - 2 = |c|$$

15. 
$$58 - 5 = e$$

**16.** 
$$58 - 0 = 56$$
 **17.**  $59 - r = 53$ 

17. 
$$59 - |r| = 53$$

18. 
$$56 - s = 52$$

Copy and complete this number pattern.

						2001	Pui
19.	_	1	2	3	4	5	6
	28	27	26	25	a	c	C
	49	48	47	n	0	r	S
	67	66	u	v	$\boldsymbol{x}$	z	a
	86	c	e	n	0	r	S

Name the rule, then complete the number pairs.

**20.** 
$$\{(27, 24), (37, 34), (47,  $\boxed{a}), (57, \boxed{c}), (77, \boxed{e}), (97, \boxed{n})\}$$$

**21.** 
$$\{(39, 32), (59, 52), (69, \boxed{0}), (79, \boxed{r}), (89, \boxed{s}), (99, \boxed{u})\}$$

**22.** 
$$\{(36, 35), (59, 58), (86,  $\overline{n}), (32, \overline{o}), (53, \overline{r}), (71, \overline{s})\}$$$

**23.** 
$$\{(26, 21), (37, 32), (49, \mathbf{u}), (68, \mathbf{v}), (87, \mathbf{x}), (95, \mathbf{z})\}$$

lame the missing addends.

**24.** 
$$47 - 5 = \overline{a}$$

**25.** 
$$86 - 4 = c$$

**24.** 
$$47 - 5 = \overline{a}$$
 **25.**  $86 - 4 = \overline{c}$  **26.**  $97 - 6 = \overline{e}$ 

**27.** 
$$67 - \mathbf{r} = 62$$

**28.** 
$$74 - \boxed{o} = 70$$

**27.** 
$$67 - \mathbf{r} = 62$$
 **28.**  $74 - \mathbf{o} = 70$  **29.**  $48 - \mathbf{s} = 41$ 

## Addition, one addend less than ten

Study this addition.

Step 1.

$$26 + 8 = (20 + 6) + 8$$
  
=  $20 + (6 + 8)$   
=  $20 + 14$   
2 tens 14 ones

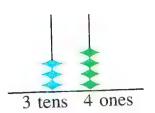
#### Step 2.

$$20 + 14 = 20 + (10 + 4)$$

$$= (20 + 10) + 4$$

$$= 30 + 4$$

$$= 34$$



#### EXERCISES

Name the sums.

$$1.20 + 15$$

7. 
$$80 + 14$$

Copy and complete these equations.

9. 
$$23 + 9 = (20 + 3) + 9$$
  
=  $20 + (3 + 9)$   
=  $20 + \boxed{0}$   
=  $\boxed{v}$ 

10. 
$$67 + 6 = (60 + 7) + 6$$
  
=  $60 + (\mathbf{n} + 6)$   
=  $60 + \mathbf{u}$   
=  $\mathbf{e}$ 

Use equations to name these sums.

11. 
$$23 + 9 = a$$

12. 
$$33 + 9 = 5$$

13. 
$$63 + 9 = \overline{n}$$

14. 
$$15 + 7 = e$$

15. 
$$14 + 8 = \mathbf{z}$$

**16.** 
$$16 + 6 = \overline{v}$$

17. 
$$59 + 3 = r$$

18. 
$$47 + 8 = \overline{u}$$

19. 
$$56 + 9 = x$$

Solve these equations.

**20.** 
$$8 + 4 = 12$$
, so  $18 + 4 = \overline{a}$ ,  $28 + 4 = \overline{s}$ ,  $78 + 4 = \overline{c}$ 

**21.** 
$$9 + 6 = 15$$
, so  $19 + 6 = x$ ,  $39 + 6 = e$ ,  $69 + 6 = r$ 

**21.** 9 + 0 = 15, so 
$$28 + 7 = 5$$
,  $48 + 7 = 48$ ,  $78 + 7 = 7$ 

**23.** 
$$9 + 3 = 12$$
, so  $29 + 3 = \overline{u}$ ,  $39 + 3 = \overline{v}$ ,  $49 + 3 = \overline{z}$ 

#### Addition, one addend less than ten





On the number line we see that 37 + 4 = 41.

#### EXERCISES

Use the number line above to name the sums and missing addends.

1. 
$$34 + 6 = a$$

2. 
$$34 + 7 = c$$

3. 
$$34 + 8 = e$$

4. 
$$36 + 6 = 0$$

5. 
$$37 + 5 = r$$

6. 
$$38 + 2 = \boxed{s}$$

7. 
$$35 + [a] = 39$$

8. 
$$35 + \boxed{c} = 41$$

9. 
$$36 + e = 41$$

10. 
$$38 + \boxed{o} = 38$$

11. 
$$39 + \mathbf{r} = 43$$

12. 
$$37 + s = 40$$

Use the number line to name the sums and missing addends.



13. 
$$55 + 6 = \boxed{c}$$

14. 
$$55 + 7 = e$$

15. 
$$55 + 9 = n$$

16. 
$$56 + 6 = 0$$

17. 
$$58 + 3 = \boxed{r}$$

18. 
$$57 + 3 = \overline{u}$$

19. 
$$59 + \mathbf{a} = 63$$

**20.** 
$$59 + \boxed{e} = 64$$

**21.** 
$$58 + n = 61$$

**22.** 
$$56 + \mathbf{0} = 60$$

**23.** 
$$58 + \mathbf{r} = 64$$

**24.** 
$$55 + [s] = 63$$

Copy and complete this number pattern.

<b>25</b> .	+	1	2	3	4	5	6	7	8	9
	37	38	39	40	a	n	C	0	e	r
	23	24	25	26	S	u	v	x	z	i
	15	16	17	18	m	w	a	C	e	n

Name the rule and then complete the number pairs.

**27.** 
$$\{(2, 7), (22, 27), (42, \boxed{0}), (52, \boxed{r}), (62, \boxed{s}), (92, \boxed{u})\}$$

**28.** 
$$\{(2, 11), (12, 21), (32, s), (42, u), (52, v), (72, s)\}$$

**29.** 
$$\{(6, 12), (16, 22), (26, \boxed{a}), (36, \boxed{n}), (56, \boxed{o}), (76, \boxed{s})\}$$

**30.** 
$$\{(4, 11), (14, 21), (24, \mathbb{Z}), (44, \mathbb{F}), (64, \mathbb{S}), (84, \mathbb{V})\}$$

#### Subtraction, one addend less than ten

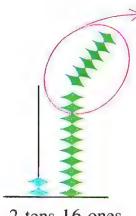
Study this subtraction.

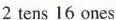
#### Step 1.

$$36 - 8 = (30 + 6) - 8$$
  
=  $(20 + 16) - 8$ 



$$(20 + 16) - 8 = 20 + (16 - 8)$$
  
=  $20 + 8$   
=  $28$ 







2 tens 8 ones

#### EXERCISES

Name the missing addends.

1. 
$$47 = 30 + a$$

**2.** 
$$56 = 40 + 0$$

**1.** 
$$47 = 30 + \boxed{a}$$
 **2.**  $56 = 40 + \boxed{o}$  **3.**  $72 = 60 + \boxed{c}$ 

4. 
$$54 = e + 14$$

**4.** 
$$54 = \boxed{e} + 14$$
 **5.**  $65 = \boxed{v} + 15$  **6.**  $88 = \boxed{z} + 18$ 

**6.** 
$$88 = \mathbf{z} + 18$$

Copy and complete the equations.

7. 
$$44 - 7 = (30 + 14) - 7$$
  
=  $30 + (14 - 7)$   
=  $30 + \overline{u}$   
=  $\overline{v}$ 

9. 
$$72 - 9 = (60 + \boxed{r}) - 9$$
  
=  $60 + (\boxed{s} - 9)$   
=  $60 + \boxed{e}$   
=  $\boxed{c}$ 

8. 
$$65 - 8 = (50 + 15) - 8$$
  
=  $50 + (\bar{a} - 8)$   
=  $50 + \bar{x}$   
=  $\bar{z}$ 

10. 
$$23 - 6 = (10 + \boxed{0}) - 6$$
  
=  $10 + (\boxed{v} - 6)$   
=  $10 + \boxed{s}$   
=  $\boxed{r}$ 

Use equations to name the missing addends.

11. 
$$43 - 7 = \overline{n}$$
 12.  $88 - 9 = \overline{c}$ 

12. 
$$88 - 9 = \overline{c}$$

13. 
$$31 - 7 = r$$

14. 
$$52 - 8 = 0$$

**14.** 
$$52 - 8 = 0$$
 **15.**  $74 - 5 = r$ 

16. 
$$95 - 7 = 0$$

17. 
$$25 - 9 = \boxed{s}$$
 18.  $41 - 8 = \boxed{u}$ 

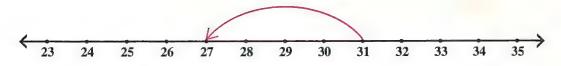
18. 
$$41 - 8 = u$$

19. 
$$57 - 8 = \boxed{z}$$

100 (one hundred)

### Subtraction, one addend less than 10





On the number line we see that 31 - 4 = 27.

### EXERCISES

Use the number line above to name the missing addends.

1. 
$$35 - 6 = \boxed{n}$$

1. 
$$35 - 6 = n$$
 2.  $35 - 7 = c$ 

3. 
$$35 - 8 = 0$$

4. 
$$35 - 9 = e$$

5. 
$$33 - 8 = 5$$

**6.** 
$$31 - 7 = v$$

Jse the number line to name the missing addends.



7. 
$$24 - 5 = a$$
 8.  $24 - 7 = c$ 

8. 
$$24 - 7 = c$$

9. 
$$24 - 8 = \boxed{s}$$

10. 
$$24 - 9 = e$$

11. 
$$21 - 4 = \mathbf{u}$$

12. 
$$23 - 6 = v$$

Copy and complete this number pattern.

13.	_	1	2	3	4	5	6	7	8	9
	35	34	33	32	a	n	C	o	e	r
	24	23	22	21	S	u	z	v	$\boldsymbol{x}$	a
	16	15	14	13	C	n	O	v	e	r

Solve the equations.

**14.** 
$$12 - 8 = 4$$
, so  $22 - 8 = \mathbf{a}$ ,  $32 - 8 = \mathbf{c}$ ,  $62 - 8 = \mathbf{e}$ 

**15.** 
$$13 - 7 = 6$$
, so  $23 - 7 = \overline{n}$ ,  $43 - 7 = \overline{0}$ ,  $73 - 7 = \overline{r}$ 

**16.** 
$$11 - 9 = 2$$
, so  $31 - 9 = 3$ ,  $41 - 9 = 4$ ,  $81 - 9 = 6$ 

17. 
$$15 - 9 = 6$$
, so  $35 - 9 = \mathbf{x}$ ,  $65 - 9 = \mathbf{z}$ ,  $95 - 9 = \mathbf{a}$ 

Name the differences.

18. 
$$23 - 8 = a$$

19. 
$$23 - 7 = c$$

**20.** 
$$35 - 7 = \mathbf{r}$$

**21.** 
$$43 - 8 = [e]$$

**22.** 
$$33 - 7 = n$$

**23.** 
$$65 - 6 = \overline{a}$$

**24.** 
$$33 - 9 = \boxed{s}$$

**25.** 
$$43 - 9 = \overline{u}$$

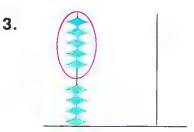
**26.** 83 
$$-4 = \boxed{z}$$

(one hundred one) 101

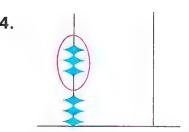
### Adding tens

Write the numeral to replace each  $\square$ .

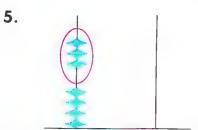
- 1.
  - 2 tens and 3 tens are **a** tens 20 + 30= e= 5020 + |c|
- 4 tens and 2 tens are c tens + 20 = e40 40 + [n]= 60



4 tens and 5 tens are n tens 40 50 + = s40 |x|= 90

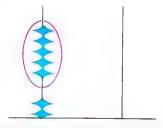


3 tens and 3 tens are o tens + 30 = [v]30 30 + z= 60



+

4 tens and 3 tens are r tens 40 + 30= [o]S = 70



2 tens and 5 tens are n tens 20 +50 = v20 + x= 70

Name the sums or missing addends.

6.

### Subtracting tens



Complete the following.

- 1. 4 sets of ten take away 2 sets of ten leaves **a** sets of ten. |e| = 2040 20
- 2. 5 sets of ten take away 3 sets of ten leaves n sets of ten.  $50 - \boxed{r} =$ 20 50 - 30
- 3. 6 sets of ten take away 2 sets of ten leaves sets of ten. v = 4060 - 20 =

Solve the equations.

**4.** 
$$60 - 30 = \boxed{a}$$
 **5.**  $70 - 20 = \boxed{c}$  **6.**  $90 - 70 = \boxed{e}$ 

**6.** 
$$90 - 70 = \boxed{e}$$

7. 
$$90 - \mathbf{n} = 40$$

**8.** 
$$70 - 10 = 0$$
 **9.**  $80 - 40 = r$ 

**10.** 
$$80 - \boxed{s} = 60$$
**13.**  $50 - 30 = \boxed{s}$ 

**11.** 
$$60 - 50 = \overline{u}$$
 **12.**  $30 - \overline{v} = 10$ 

**13.** 
$$50 - 30 = \boxed{x}$$
 **14.**  $90 - \boxed{z} = 20$  **15.**  $100 - 40 = \boxed{a}$ 

Name the missing addends.

23. 80 24. 80 25. 90 26. 70 27. 40 28. 60 29. 100 
$$\frac{-a}{60}$$
  $\frac{-c}{30}$   $\frac{-e}{50}$   $\frac{-30}{n}$   $\frac{-10}{0}$   $\frac{-r}{20}$   $\frac{-50}{s}$ 

Solve the problems.

- 30. There are 80 yellow and blue books on the top shelf of the book case. If 30 of the books are yellow, how many books are blue?
- On the bottom shelf there are 60 books about arithmetic and history. If 20 of the books are about arithmetic, how many are about history?
- **32.** On the two shelves there are 90 books. If there are 60 books on the bottom shelf, how many are there on the top shelf?



### Adding and subtracting tens

Name the sums.

Name the missing addends.

Complete the number pairs.

**13.** 
$$\{(20, 40), (30, 50), (40, \boxed{a}), (60, \boxed{c}), (80, \boxed{e})\}$$

**14.** 
$$\{(70, 30), (50, 10), (80, \boxed{n}), (60, \boxed{o}), (90, \boxed{r})\}$$



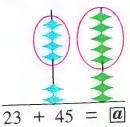
- **15.** Bobby had 2 strikes and then 3 more strikes. How many pins did he knock down with 5 strikes?
- 16. John had 6 strikes, Larry had 4. How many more pins than Larry did John knock down?
- 17. Marilyn scored 30 points and Ellen scored 70 points. How many more points did Ellen score?
- 18. Rita and Sue scored 80 points together. Pat scored 40 points. What was the total score for the three girls?



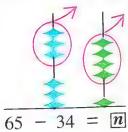
- 1. There are 20 peanut butter and 30 chocolate chip cookies on one tray. How many cookies are there in all?
- 2. One tray has 30 ginger bread men with yellow buttons and 40 ginger bread men with blue buttons. How many ginger bread men are there in all?
- **3.** In the morning there were 90 cookie stars. By afternoon 30 had been sold. How many cookie stars were left?
- **4.** One tray contains 10 white cupcakes and 30 chocolate cupcakes. How many cupcakes are there in all?
- **5.** There are 80 ginger snap cookies, but 40 are broken. How many cookies are not broken?
- **6.** A small tray can hold 3 rows of 10 cookies. How many cookies can it hold?
- 7. A large tray can hold 10 cookies in one row. If it has 4 rows of oatmeal cookies and 5 rows of sugar cookies, how many cookies does it hold?

Use the abacus to help you solve each equation.

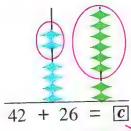
1.



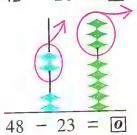
4.



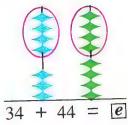
2.



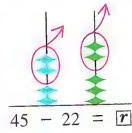
5.



3.



6.



Write the numeral to complete the sentence, then solve the equation.

7. 4 tens and 4 ones joined with 5 tens and 3 ones forms a tens and n ones.

$$44 + 53 = C$$

8. 3 tens and 2 ones taken from 5 tens and 6 ones leaves stens and ones.

$$56 - 32 = v$$

9. 4 tens and 2 ones taken from 7 tens and 8 ones leaves  $\overline{r}$  tens and  $\overline{x}$  ones.

$$78 - 42 = e$$

Name the sums or differences.

11. 3 tens 5 ones 
$$+2$$
 tens 4 ones  $\overline{u}$  tens  $\overline{v}$  ones

14. 7 tens 8 ones 
$$-6$$
 tens 2 ones  $\boxed{n}$  tens  $\boxed{s}$  ones



Study these examples.

$$\begin{array}{r}
23 = 20 + 3 \\
+ 45 = + 40 + 5 \\
\hline
60 + 8 = 68
\end{array}$$

$$\begin{array}{r}
65 = 60 + 5 \\
-34 = -30 - 4 \\
\hline
30 + 1 = 31
\end{array}$$

EXERCISES

Copy and complete.

Name the sum or missing addend using the method above.

Solve these problems.

- **23.** Bruce took an arithmetic test that had 23 addition and 35 subtraction problems on it. How many problems was this in all?
- **24.** Tony took an arithmetic test that had 39 problems on it. He had 12 wrong. How many problems did he have right?

Study these 3-step methods.

### Addition

9 Step 1: 
$$3 + 6 = 9$$

$$50$$
 Step 2:  $20 + 30 = 50$ 

59 Step 3: 
$$50 + 9 = 59$$

### Subtraction

5 Step 1: 
$$8 - 3 = 5$$

$$40 \quad Step \ 2: \ 60 \ -20 \ = 40$$

$$45$$
 Step 3:  $40 + 5 = 45$ 

51

+ 46

+ 22

### EXERCISES

Use the 3-step method to name the sums.

+ 34

+ 11

+46

+ 21

+ 24

Use the 3-step method to name the missing addends.

Solve the problems.

- 31. There are 56 children in the first grade and 43 in the second grade. How many more children are in the first grade?
- **32.** There are 89 children in the sixth grade and 65 in the third grade. How many more children are in the sixth grade?

108 (one hundred eight)



Study these 2-step methods.

Addition  23 $+ 36$ Step 1: $3 + 6 = 9$ $59$ Step 2: $20 + 30 = 50$	Subtraction  68  - 23 Step 1: 8 - 3 = 5  45 Step 2: 60 - 20 = 40
---	--

### EXERCISES

Use the 2-step method to name the sums or missing addends.

1. 
$$\frac{14}{+25}$$
2.  $\frac{81}{+17}$ 3.  $\frac{63}{+24}$ 4.  $\frac{54}{+23}$ 5.  $\frac{82}{+15}$ 6.  $\frac{20}{+37}$ 7.  $\frac{46}{-21}$ 8.  $\frac{87}{-26}$ 9.  $\frac{37}{-14}$ 10.  $\frac{58}{-23}$ 11.  $\frac{39}{-26}$ 12.  $\frac{86}{-40}$ 

Name the sums or missing addends.

13. 
$$36$$
 14.  $26$  15.  $24$  16.  $61$  17.  $25$  18.  $34$   $+ 42$   $+ |r|$  48  $+ 35$   $|r|$  61  $|r|$  77  $|r|$  19.  $76$  20.  $38$  21.  $75$  22.  $36$  23.  $32$  24.  $63$   $- 34$   $|r|$   $|r$ 

Solve the problems.

- 25. The children in the Riverside School made gift packages to send to needy children. One class made 31 and another 42. How many packages did they make in all?
- **26.** The third grade made 58 packages and the second grade 43. How many more packages did the third grade make?
- 27. The fifth grade made 75 packages and the first grade made 32. How many more packages did the fifth grade make?

# Adding and subtracting hundreds, tens, and ones

Write the numeral to replace each  $\square$ .

1. 3 hundreds plus 4 hundreds =  $\boxed{a}$  hundreds

$$300 + 400 = \boxed{c}$$
  
 $300 + \boxed{r} = 700$ 

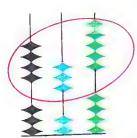
2. 8 hundreds minus 2 hundreds =  $\boxed{c}$  hundreds

$$800 - 200 = \mathbf{n}$$

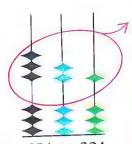
$$800 - \mathbf{r} = 600$$

Name the sums or missing addends.

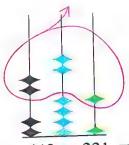
Name the sums or missing addends.



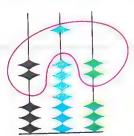
8. 
$$324 + 435 = a$$



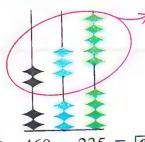
9. 
$$\overline{654 - 321} = \boxed{c}$$



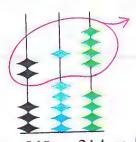
10. 
$$462 - 221 = e$$



11. 
$$\overline{373} + 212 = \mathbf{n}$$



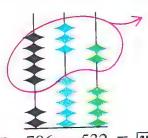
12. 
$$469 - 235 = e$$



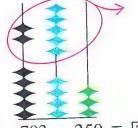
13. 
$$568 - 214 = \mathbf{r}$$



14. 
$$\overline{526} - 312 = S$$



15. 
$$\overline{786 - 532} = u$$



16. 
$$\overline{793} - 250 = x$$

# Adding and subtracting hundreds, tens, and ones



Study these examples.

### Addition

$$324 = 300 + 20 + 4$$

$$+ 435 = 400 + 30 + 5$$

$$700 + 50 + 9 = 759$$

### Subtraction

$$567 = 500 + 60 + 7
-432 = -400 - 30 - 2$$

$$100 + 30 + 5 = 135$$

### EXERCISES

Copy and complete each example.

Name the sums or missing addends.

olve this problem.

**20.** At the book fair, the third grade bought 103 books, and the fourth grade bought 126. How many more books did the fourth grade buy than the third?



# Adding and subtracting hundreds, tens, and ones

These examples are like those on page 108 but they have four steps. Can you see why?

### Addition

### **Subtraction**

### EXERCISES

Use the 4-step method to name the sums.

Use the 4-step method to name the missing addends.

Solve this problem.

21. The children in the third and fourth grades went to the book fair. If there are 137 children in the third grade, and 152 in the fourth grade, how many children went to the fair?

112 (one hundred twelve)

### Three-place addition and subtraction



Study these examples.

### Addition

### **Subtraction**

H stands for Hundreds, T for Tens, O for Ones

### EXERCISES

Use the column method to name the sums or missing addends.

Study these examples of the short method.

$$\begin{array}{rrr}
464 & 658 \\
+ 231 & - 321 \\
\hline
695 & 337
\end{array}$$

Jse the short method to name the sums or missing addends.

Jse the short method to name the sums and missing addends in Exercises 1–20 on page 112. Use the answers from the 4-step method to check your vork.

### Adding tens and ones

We may name the sum of 37 and 25 by the methods we used on pages 98 and 107.

$$37 = 30 + 7 
+ 25 = 20 + 5 
\hline
50 + 12 = (50 + 10) + 2 = 62$$

By now you can probably rename 50 + 12 as 62 in one step, so you will be able to write your work out like this:

$$\begin{array}{r}
 37 = 30 + 7 \\
 + 25 = 20 + 5 \\
 \hline
 50 + 12 = 62
 \end{array}$$

### EXERCISES

Copy and complete the equations.

Use expanded numerals to name these sums.

### Adding tens and ones



We can use the 3-step method to name the sum 37 + 25.

$$\begin{array}{r}
37 \\
+ 25 \\
\hline
12 \\
50 \\
\hline
62
\end{array}$$

$$Step 1: 7 + 5 = 12 \\
Step 2: 30 + 20 = 50 \\
Step 3: 50 + 12 = 62$$

### EXERCISES

Use the 3-step method to name the sums.

solve these problems.

- 19. On First Street David delivers 27 papers; on Second Street he delivers 38 papers. How many is that in all?
- **20.** There were 27 cars in front of the station and 36 cars behind. How many cars in all?
- 21. In the morning the temperature was 35 degrees. During the afternoon it rose 26 degrees. What was the temperature then?

## Adding tens and ones

Chuck wanted to use a shorter method than the one on the last page.

Chuck wrote:

Chuck thought:

$$\frac{1}{37} + 25 = 62$$

$$7 + 5 = 12$$

Write 2 in the ones' place.

$$1 + 3 + 2 = 6$$

Write 6 in the tens' place.

### EXERCISES

Chuck's method is the regular short method. Use it to name these sums.

1. 
$$32$$
 2.  $46$  3.  $48$  4.  $53$  5.  $64$  6.  $36$  7.  $65$   $+ 19$   $+ 27$   $+ 17$   $+ 36$   $+ 15$ 

Use the short method to name the sums.

**15.** 
$$23 + 49 = a$$
 **16.**  $47 + 28 = c$ 

$$16. 47 + 28 = C$$

17. 
$$67 + 29 = C$$

65

7.

18. 
$$46 + 28 = 0$$

19. 
$$27 + 59 = r$$

**20.** 
$$14 + 48 = \boxed{s}$$

We can use the short method with hundreds, tens, and ones. Study this example.

Write 3 in the ones' place.

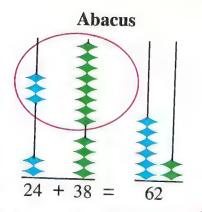
Write 7 in the tens' place.

Write 3 in the hundreds' place.

Use the short method to name the sums.

# Adding tens and ones Working form

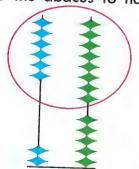




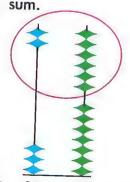
$$\begin{array}{r}
 1 \\
 24 \\
 + 38 \\
 \hline
 62
 \end{array}$$

### EXERCISES

Use the abacus to name each sum.



1. 
$$25 + 68 = a$$

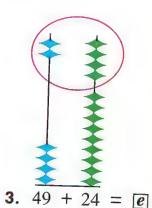


**2.** 
$$37 + 26 = c$$

7.

49

+ 22



lame the sums.

27

+39

10.

27

se the working form to name the sums.

**22.** 
$$23 + 39 = \overline{a}$$
 **25.**  $37 + 24 = \overline{c}$ 

**23.** 
$$54 + 26 = \mathbf{c}$$

**26.** 
$$14 + 59 = e$$

**28.** 
$$33 + 58 = e$$

**29.** 
$$26 + 59 = u$$

**31.** 
$$64 + 29 = \mathbf{n}$$
 **32.**  $16 + 79 = \mathbf{v}$ 

**24.** 
$$64 + 26 = \mathbf{c}$$

**27.** 
$$27 + 48 = e$$

**30.** 
$$46 + 35 = \overline{n}$$

33. 
$$75 + 18 = r$$

## Subtracting tens and ones

To name the difference 63 - 28 we may set up our work like this:

$$\begin{array}{rcl}
63 & = & 60 + 3 \\
-28 & = & -20 - 8
\end{array}$$

Do you see the problem? Can you subtract 8 from 3 and come up with a number like the numbers you are using? No? Is 8 greater than 3?

How can we solve this problem? Let us rename 63 so that we can subtract in the ones' place. Pick one of these other names for 63: 40 + 23, 30 + 33, 50 + 13. Did you pick 50 + 13? Now let us try again.

### EXERCISES

Copy and complete these examples.

Use expanded numerals to name the missing addends.

11. 62 12. 33 13. 54 14. 61 15. 84 16. 63 17. 80 
$$-18$$
  $-17$   $-28$   $-35$   $-27$   $-37$   $-39$ 

### Subtracting tens and ones



Betty and Bob wanted to use shorter methods than the one on the last page. Compare their methods.

Betty's

Bob's

Both methods use the same steps.

$$\begin{array}{r}
 50 & 13 \\
 63 = \cancel{60} + \cancel{3} \\
 -28 = -20 - 8 \\
 \hline
 30 + 5 = 35
 \end{array}$$

$$\begin{array}{r}
 5 & 13 \\
 \underline{63} \\
 -28 \\
 \hline
 35
 \end{array}$$

Step 2: 
$$13 - 8 = 5$$

Step 3: 
$$50 - 20 = 30$$

### EXERCISES

Bob's method is the regular short method. Use the regular short method to name these missing addends.

45

We can use the short method with hundreds, tens, and ones. Study this example.

Step 1: Rename 
$$572$$
 as  $500 + 60 + 12$ 

Step 2: 
$$12 - 6 = 6$$

Step 3: 
$$60 - 30 = 30$$

Step 4: 
$$500 - 200 = 300$$

Jse the short method to name the missing addends.

### Subtracting tens and ones

### Column method

Tens	Ones
3 4 - 2	12 2 5
1	7

### Short method

$$\begin{array}{c|c}
 & 3 & 12 \\
 & 2 & 2 \\
 \hline
 & 2 & 5 \\
\hline
 & 1 & 7
\end{array}$$

### EXERCISES

Use the column method to name these differences.

Use the short method to name these differences.

92

Solve this problem.

- 41. If 28 of 57 children are boys, how many are girls?
- 120 (one hundred twenty)

### Practice page



Name the sums or missing addends.

Complete these number sentences by writing <, >, or =, to replace each  $\bigcirc$ .

**33.** 
$$17 + 9$$
 ()  $18 + 8$ 

Name the sums or missing addends.

### How to solve story problems

Step 1. Think: What are the sets?

Step 2. Think: What is happening to the sets?

Step 3. Write an equation.

Step 4. Solve the equation.

Step 5. Use the number to solve the problem.

### EXERCISES

Use the 5-step method to solve these problems.

- 1. There were 30 children on the swings and 10 playing baseball. How many children were there in all on the playground?
- 2. There were 27 girls and 16 boys in a large circle. How many boys and girls were there in all?
- **3.** If you had 46 baseball cards and collected 23 more, how many baseball cards would you have then?
- 4. Madge is 45 inches tall and Beth is 43 inches tall. How much taller is Madge?
- 5. A certain television show is 45 minutes long. If 29 minutes of the show have passed, how many minutes are left?
- 6. There are 35 students in one class and 37 in the class next door. How many students are there in all?

### CHECKING UP

Copy and complete these equations.

1. 
$$14 + 5 = (10 + 4) + 5$$
  
=  $10 + (e + a)$   
=  $10 + r$   
=  $s$ 

2. 
$$25 + 7 = (20 + 5) + 7$$
  
=  $20 + (s + w)$   
=  $20 + r$   
=  $e$ 

Complete these equations.

3. Since 
$$3 + 5 = 8$$
,  $13 + 5 = \overline{a}$ ,  $23 + 5 = \overline{r}$ ,  $53 + 5 = \overline{n}$ 

**4.** Since 
$$7 + 2 = 9$$
,  $17 + 2 = \overline{a}$ ,  $27 + 2 = \overline{s}$ ,  $47 + 2 = \overline{c}$ 

**5.** Since 
$$3 + 6 = 9$$
,  $13 + 6 = \overline{v}$ ,  $23 + 6 = \overline{e}$ ,  $63 + 6 = \overline{u}$ 

**6.** Since 
$$8 + 6 = 14$$
,  $28 + 6 = \overline{u}$ ,  $48 + 6 = \overline{r}$ ,  $88 + 6 = \overline{n}$ 

7. Since 
$$7 + 4 = 11$$
,  $57 + 4 = 3$ ,  $77 + 4 = 6$ ,  $87 + 4 = 6$ 

**8.** Since 
$$8 + 9 = 17$$
,  $38 + 9 = 2$ ,  $48 + 9 = 7$ ,  $68 + 9 = 6$ 

Name the sums.

**9.** 
$$50 + 14$$
 **10.**  $40 + 15$  **11.**  $60 + 17$  **12.**  $20 + 18$ 

Name the missing addends.

Name the sums or missing addends.

27. 60 28. 46 29. 57 30. 
$$\boxed{z}$$
 31. 62 32. 38 33. 49  $+ 40$   $\boxed{r}$  89 55  $\boxed{e}$   $\boxed{s}$  26

Name the sums or differences.

### LOOKING BACK

By the Grouping Rule we know that 
$$14 + 8 = (10 + 4) + 8$$
  
=  $10 + (4 + 8)$   
=  $10 + 12$   
=  $22$ 

Use the grouping rule to name these sums.

1. 
$$37 + 4$$

$$3.39 + 1$$

We can use expanded numerals, a 3-step method, or a short method to add or subtract tens and ones.

Expanded numerals	3-step method	Short method
$     \begin{array}{r}       27 = 20 + 7 \\       + 45 = +40 + 5 \\       \hline       60 + 12 = 72     \end{array} $	$   \begin{array}{r}     27 \\     + 45 \\     \hline     12 \\     \hline     60 \\     \hline     72   \end{array} $	1 27 + 45 72
$ \begin{array}{r} 30 + 15 \\ 45 = 40 + 5 \\ -27 = -20 - 7 \\ \hline 10 + 8 = 18 \end{array} $	$ \begin{array}{r}     315 \\     45 \\     -27 \\     \hline     8 \\     10 \\     \hline     18 \end{array} $	3 15 45 -27 18

Use the 3-step method and then the short method to name the sums.

Use the short method to name the differences.

### EXTRA FOR EXPERTS



Addition In Your Head

To add 56 and 23, think: Step 1: 56 Step 2: 76 
$$\frac{+20}{76}$$
  $\frac{+3}{79}$ 

Try this method to name the sums.

Make up other examples, name the sum by this method and have your neighbor check your work.

Subtraction In Your Head

To subtract 32 from 75, think: Step 1: 
$$75$$
  $-30$   $-2$   $45$ 

ry this method to name the missing addends.

ake up other subtraction examples. Name the missing Idends and have your neighbor check your work.

# CHAPTER 5 • Multiplication and Division with Factors to 5

### Two as a factor

$$2 + 2 + 2 = 6$$

You write "2" 3 times.

$$3 \times 2 = 6$$

3 times 2 is 6.

### EXERCISES

Use the rods to solve the equations.







1. 
$$2 + 2 + 2 = a$$

$$3 \times 2 = \mathbf{u}$$

$$3 + 3 = C$$

$$2 \times 3 = v$$

**2.** 
$$2 + 2 + 2 + 2 = e$$

$$4 \times 2 = \mathbf{r}$$

$$4 + 4 = s$$

$$2 \times 4 = x$$

1. 
$$2+2+2=a$$
 2.  $2+2+2+2=e$  3.  $2+2+2+2+2=n$ 

$$5 \times 2 = 0$$

$$5 + 5 = a$$

$$2 \times 5 = \boxed{c}$$





- 4.  $6 \times 2 = n$ 6 + 6 = x
  - $2 \times 6 = 0$

5.  $7 \times 2 = 0$ 7 + 7 = z $2 \times 7 = \mathbf{r}$ 





- 6.  $8 \times 2 = s$  $8 + 8 = \boxed{e}$ 
  - $2 \times 8 = \boxed{u}$

7.  $9 \times 2 = v$ 9 + 9 = 0 $2 \times 9 = \boxed{x}$ 

### Counting by twos



$$2 + 2 + 2 = 6$$
  
3 addends of 2.  
The sum is 6.

$$3 \times 2 = 6$$
  
The factors are 3 and 2.  
The product is 6.

### EXERCISES



Use the number line to complete this set.

Count by twos on the number line to rename these products.

3. 
$$2 \times 2$$

**5.** 
$$4 \times 2$$

8. 
$$7 \times 2$$

11. 
$$10 \times 2$$

**12.** 
$$11 \times 2$$

13. 
$$12 \times 2$$

Name the number on the number line just after:

14. 
$$3 \times 2$$

**15.** 
$$4 \times 2$$
 **16.**  $5 \times 2$ 

16. 
$$5 \times 2$$

17. 
$$7 \times 2$$

**18.** 
$$1 \times 2$$

**21.** 
$$9 \times 2$$

Name the number on the number line just before:

**22.** 
$$4 \times 2$$

**27.** 
$$6 \times 2$$
 **28.**  $8 \times 2$ 

**28.** 
$$8 \times 2$$

Name the number on the number line between:

**30.** 
$$3 \times 2$$
 and  $4 \times 2$ 

31. 
$$5 \times 2$$
 and  $6 \times 2$ 

**32.** 
$$6 \times 2$$
 and  $7 \times 2$ 

**33.** 
$$8 \times 2$$
 and  $9 \times 2$ 

**34.** 
$$4 \times 2$$
 and  $5 \times 2$ 

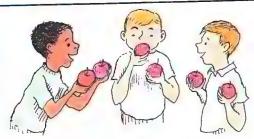
**35.** 
$$9 \times 2$$
 and  $10 \times 2$ 

lame the products.

**44.** 
$$2 \times 3$$

**44.** 
$$2 \times 3$$
 **45.**  $5 \times 2$ 

### Factors and products



How many boys can have 2 apples each?

$$\begin{array}{c} \boxed{\alpha} \times 2 = 6 \\ 3 \times 2 = 6 \end{array}$$





How many apples for each boy?

$$2 \times \boxed{a} = 6$$
$$2 \times 3 = 6$$

The **missing factor** is 3. The factor you know is 2. The product is 6.

### EXERCISES

Name the products.

$$1. \ 2 \times 3 = \boxed{a}$$

4. 
$$8 \times 2 = e$$

**2.** 
$$2 \times 5 = s$$

5. 
$$9 \times 2 = r$$

3. 
$$2 \times 7 = C$$

**6.** 
$$6 \times 2 = \bar{n}$$

Name the missing factors.

7. 
$$2 \times a = 6$$

10. 
$$e \times 2 = 18$$

13. 
$$2 \times n = 8$$

16. 
$$x \times 2 = 16$$

19. 
$$a \times 2 = 2 \times 3$$

**22.** 
$$e \times 2 = 2 \times 9$$

**25.** 
$$o$$
 × 2 = 2 × 7

8. 
$$2 \times s = 10$$

11. 
$$r \times 2 = 12$$

**14.** 
$$\mathbf{0} \times 2 = 10$$

17. 
$$2 \times \mathbf{z} = 12$$

**9.** 
$$2 \times c = 14$$

12. 
$$2 \times \bar{u} = 4$$

15. 
$$v \times 2 = 2$$

18. 
$$2 \times s = 18$$

19. 
$$a \times 2 = 2 \times 3$$

**20.** 
$$o \times 5 = 5 \times 2$$

**23.** 
$$8 \times 2 = 2 \times \boxed{s}$$

**26.** 
$$1 \times 2 = 2 \times v$$

**21.** 
$$4 \times e = 2 \times 4$$

**24.** 
$$2 \times 6 = \mathbf{n} \times 2$$

**27.** 
$$9 \times 2 = n \times 9$$

Name the missing factors.

28. 
$$4 \times \boxed{a} \times \boxed{z} \times \boxed{z}$$

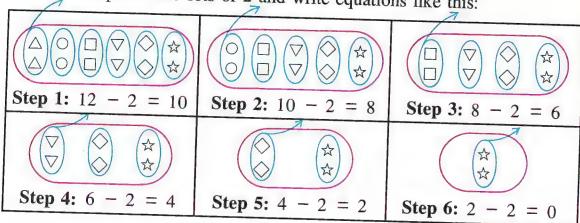
30. 
$$\frac{c}{\times 2}$$

$$\frac{2}{\sqrt{n}}$$

### How many sets of 2?



To find the number of sets of 2 in a set of 12 we can separate the sets of 2 and write equations like this:



We took away a set of 2 six times. There are 6 sets of 2 in a set of 12. We may write the equations:  $n \times 2 = 12 \text{ or } 12 \div 2 = n$ .

The equation  $12 \div 2 = n$  may be read "How many twos are there in twelve?" The missing factor is 6.

### EXERCISES

Use the sets to solve these equations.

- 1. 00
  - $4 \div 2 = \boxed{a}$
- 000
  - $6 \div 2 = |c|$
- 0000 0000
  - $8 \div 2 = [e]$

- 000000 00000
  - $12 \div 2 = \boxed{e}$
- 0000000
  - $14 \div 2 = [r]$
- - $16 \div 2 = |s|$

Write the numeral to complete each sentence.

- **7.** 6 sets of 2 form 1 set of **a**.
- **9.** 5 sets of 2 form 1 set of **C**.
- 11. 3 sets of 2 form 1 set of **@**.
- **8.** 1 set of 12 forms  $\bar{x}$  sets of 2.
- **10.** 1 set of 10 forms **2** sets of 2.
- 12. 1 set of 6\forms  $\overline{n}$  sets of 2.
- 13. 8 sets of 2 form 1 set of v. 14. 1 set of 16 forms v sets of 2.

(one hundred twenty-nine) 129

### Division with two as a factor

factor 
$$\times$$
 factor = product  
 $3 \times 2 = 6$ 

Division is the opposite of multiplication.

### EXERCISES

Solve these equations.

1. 
$$2 \times 3 = 6$$
, so  $6 \div 3 = \mathbf{a}$ 

3. 
$$2 \times 5 = 10$$
, so  $10 \div 2 = \boxed{c}$ 

5. 
$$7 \times 2 = 14$$
, so  $14 \div 2 = \boxed{e}$ 

**2.** 
$$2 \times 4 = 8$$
, so  $8 \div 4 = n$ 

**4.** 
$$6 \times 2 = 12$$
, so  $12 \div 6 = \boxed{0}$ 

**6.** 
$$9 \times 2 = 18$$
, so  $18 \div 9 = \boxed{r}$ 

Name the missing factors.

7. 
$$a \times 2 = 2$$

8. 
$$c \times 2 = 6$$

9. 
$$5 \times \bar{u} = 10$$

10. 
$$2 \times \bar{n} = 10$$

11. 
$$\mathbf{z} \times 2 = 12$$

12. 
$$2 \times v = 8$$

13. 
$$2 \times x = 16$$

14. 
$$e \times 2 = 18$$

15. 
$$2 \times a = 12$$

Name the missing factors.

16. 
$$6 \div 2 = a$$

17. 
$$10 \div 2 = r$$

18. 
$$12 \div 2 = C$$

19. 
$$14 \div 2 = e$$

**20.** 
$$16 \div 2 = n$$

**21.** 
$$18 \div 2 = \boxed{s}$$

**22.** 
$$10 \div \overline{u} = 2$$

23. 
$$12 \div 2 = n$$

**24.** 
$$8 \div a = 2$$

**25.** 
$$8 \div 2 = x$$

**26.** 
$$\mathbf{z} \div 2 = 4$$

**27.** 
$$14 \div \mathbf{r} = 2$$

Name the missing factors.

28. 
$$a$$
 29.  $n$  30.  $\times 2$ 

$$\frac{|c|}{\times 2}$$

$$\frac{[c]}{18}$$
 31.  $\frac{[r]}{\times 2}$  32.  $\frac{2}{12}$ 

33. 
$$2 \times \boxed{\$}$$

34. 
$$\frac{2}{\times r}$$

Name the missing factors.

130 (one hundred thirty)

### How many in each set?

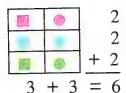


### Complete the matchings.

If **Then** 1. 1 **2**. 1 5 |z|1 1 1

# Three as a factor

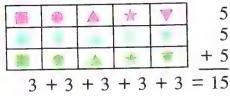
Use the patterns to solve the equations.



	1	A	*	4
			*	4
2357 Edie		A	女	+ 4
3 +	3 -	+ 3	+ 3	= 12

1. 
$$2 + 2 + 2 = \mathbf{a}$$
  
 $3 \times \mathbf{c} = 6$   
 $3 + 3 = \mathbf{e}$   
 $2 \times \mathbf{n} = 6$ 

2. 
$$3 + 3 + 3 = \boxed{e}$$
 3.  $4 + 4 + 4 = \boxed{o}$   $3 \times \boxed{r} = 12$   $3 + 3 + 3 + 3 = \boxed{s}$   $4 \times \boxed{u} = 12$ 



4. 
$$5 + 5 + 5 = \boxed{a}$$
  
 $3 \times 5 = \boxed{c}$   
 $3 + 3 + 3 + 3 + 3 = \boxed{e}$   
 $5 \times 3 = \boxed{n}$ 

$$\begin{array}{c}
 6 + 6 + 6 &= \boxed{0} \\
 3 \times 6 &= \boxed{r} \\
 3 + 3 + 3 + 3 + 3 &= \boxed{s} \\
 6 \times 3 &= \boxed{u}
 \end{array}$$

In this pattern we see that 6 sets of 3 may be joined to form 1 set of 18.



5.

Use the pattern to solve these equations.

6. 
$$7 \times 3 = a$$

7. 
$$3 \times 7 = c$$

8. 
$$3 \times e = 21$$

9. 
$$8 \times 3 = x$$

10. 
$$3 \times 8 = T$$

11. 
$$3 \times s = 24$$

12. 
$$9 \times 3 = v$$

13. 
$$3 \times 9 = x$$

14. 
$$3 \times \mathbf{z} = 27$$

15. 
$$7 \times \bar{n} = 21$$

16. 
$$8 \times \bar{u} = 24$$

17. 
$$9 \times a = 27$$

### Counting by threes



$$3 + 3 + 3 + 3 = 12$$

4 addends of 3.

The sum is 12.

 $4 \times 3 = 12$ 

The factors are 4 and 3.

The product is 12.





Jse the number line to complete this set.

Jse the number line to rename these products.

**2.** 
$$2 \times 3$$

**4.** 
$$4 \times 3$$

**5.** 
$$5 \times 3$$

8. 
$$8 \times 3$$

Name the number on the number line just after:

11. 
$$5 \times 3$$

12. 
$$4 \times 3$$

15. 
$$2 \times 3$$

Name the number on the number line just before:

18. 
$$4 \times 3$$

19. 
$$6 \times 3$$

**20.** 
$$7 \times 3$$

**21.** 
$$2 \times 3$$

**24.** 
$$5 \times 3$$

Name the numbers on the number line between:

**26.** 
$$1 \times 3$$
 and  $2 \times 3$ 

**27.** 
$$3 \times 3$$
 and  $4 \times 3$ 

**28.** 5 
$$\times$$
 3 and 6  $\times$  3

**29.** 
$$7 \times 3$$
 and  $8 \times 3$ 

**30.** 
$$7 \times 3$$
 and  $6 \times 3$ 

31. 
$$9 \times 3$$
 and  $8 \times 3$ 

Name the products or missing factors.

3

3

r

39.

34.

42. 
$$3 \times e \over 9$$

43. 
$$\frac{3}{\times 5}$$

44. 
$$r$$
  $\times$  3

45. 
$$\frac{u}{\times 3}$$

### Multiplication and division with three

1	2	3	4	5	6	7	8	9
gen sayang	•	<u>***</u> :	1	W	<b>♦</b>	-	C	8
		_	*	_	<b>♦</b>	-	C	*
		_	*	<b>V</b>	•	-	C	8
3	6	9	12	15	18	21	24	27

### EXERCISES

Write the numerals to complete the sentences.

- 1. 3 sets of 3 form a set of  $\overline{a}$ .
- 2. A set of 9 forms e sets of 3.
- 3. A set of 15 forms c sets of 3.
- **4.** 6 sets of 3 form a set of  $\overline{n}$ .
- **5.** 8 sets of 3 form a set of **a**.
- 6. A set of 24 forms v sets of 3.
- 7. A set of 27 forms **z** sets of 3.
- **8.** A set of 6 forms 3 sets of [u].
- 9. 3 sets of 4 form a set of r.
- 10. A set of 18 forms 3 sets of S.
- 11. 3 sets of 7 form a set of  $\bar{x}$ .
- **12.** A set of 27 forms 3 sets of **C**.

Use the pattern at the top of the page to solve the equations.

13. 
$$a \times 3 = 9$$

$$3 \times e = 9$$

$$9 \div 3 = \boxed{c}$$

$$9 \div \mathbf{n} = 3$$

14. 
$$\boxed{o} \times 3 = 12$$

$$3 \times |\overline{r}| = 12$$

$$12 \div 3 = \boxed{s}$$

$$12 \div \mathbf{u} = 3$$

15. 
$$v \times 3 = 15$$

$$3 \times \boxed{x} = 15$$

$$15 \div 3 = \mathbf{z}$$

$$15 \div e = 3$$

16. 
$$a \times 3 = 21$$

$$3 \times \mathbf{n} = 21$$

$$21 \div 3 = \boxed{o}$$

$$21 \div \boxed{r} = 3$$

17. 
$$[s] \times 3 = 24$$

$$3 \times \mathbf{c} = 24$$

$$24 \div \boxed{v} = 3$$

18. 
$$\mathbf{z} \times 3 = 27$$

$$3 \times [n] = 27$$

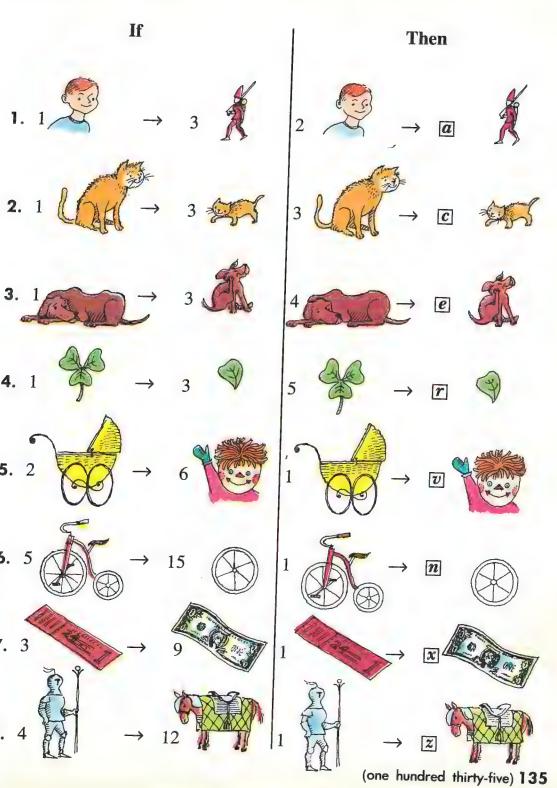
$$24 \div 3 = \mathbf{u}$$
$$27 \div 3 = \mathbf{s}$$

$$27 \div [a] = 3$$

# How many in each set?

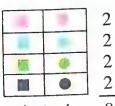


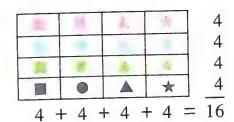
### Complete the matchings.



### Four as a factor

Use the patterns to solve the equations.





$$4 + 4 = 8$$
  $4$   
1.  $2 + 2 + 2 + 2 = \boxed{a}$ 

$$4 \times \mathbf{u} = 8$$

$$4 + 4 = \mathbf{c}$$

$$2 \times \mathbf{v} = 8$$

2. 
$$3 + 3 + 3 + 3 = \boxed{e}$$
  
 $4 \times \boxed{a} = 12$   
 $4 + 4 + 4 = \boxed{r}$   
 $3 \times \boxed{n} = 12$ 

3

3

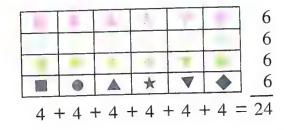
3

3

3. 
$$4 + 4 + 4 + 4 = \boxed{r}$$
  
 $4 \times \boxed{e} = 16$ 



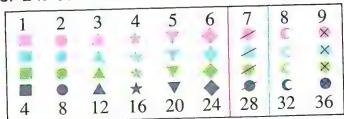
4 + 4 + 4 + 4 + 4 = 20  
4. 
$$5 + 5 + 5 + 5 = \boxed{u}$$
  
 $4 \times \boxed{s} = 20$   
 $4 + 4 + 4 + 4 + 4 = \boxed{x}$ 



5. 
$$6+6+6+6=0$$
  
 $4 \times a = 24$   
 $4+4+4+4+4=2$   
 $6 \times r = 24$ 

In this pattern we see that 6 sets of 4 may be joined to form 1 set of 24. So 7 sets of 4 form 1 set of 28.

 $5 \times \mathbf{z} = 20$ 



Use the pattern to solve the equations.

6. 
$$4 \times 7 = C$$

$$7 \times 4 = a$$

$$4 \times \mathbf{e} = 28$$

$$7 \times \mathbf{n} = 28$$

7. 
$$4 \times 8 = r$$

$$8 \times 4 = 0$$

$$8 \times \boxed{s} = 32$$

$$4 \times \mathbf{u} = 32$$

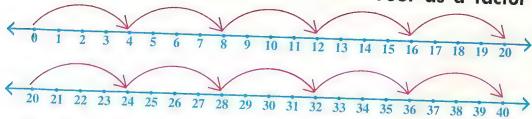
8. 
$$4 \times 9 = x$$

$$9 \times 4 = v$$

$$9 \times \mathbb{Z} = 36$$

$$4 \times \boxed{a} = 36$$





Use the number line to complete this set:

Use the number line to name these products.

**2.** 
$$2 \times 4$$

**3.** 
$$3 \times 4$$

**4.** 
$$5 \times 4$$

**5**. 
$$6 \times 4$$

Name the number on the number line just after:

11. 
$$4 \times 4$$

12. 
$$5 \times 4$$

14. 
$$7 \times 4$$

16. 
$$6 \times 4$$

Name the number on the number line just before:

18. 
$$2 \times 4$$

19. 
$$4 \times 4$$

**21.** 
$$7 \times 4$$

**22.** 
$$5 \times 4$$

lame the numbers on the number line between:

**26.** 
$$1 \times 4$$
 and  $2 \times 4$ 

**27.** 
$$3 \times 4$$
 and  $4 \times 4$ 

**28.** 5 
$$\times$$
 4 and 6  $\times$  4

**29.** 
$$7 \times 4$$
 and  $6 \times 4$ 

**30.** 
$$8 \times 4$$
 and  $9 \times 4$ 

31. 
$$3 \times 4$$
 and  $2 \times 4$ 

ame the products.

ame the missing factors.

40. 
$$C$$
 41.  $\frac{\times 4}{24}$ 

$$\frac{n}{36}$$

43. 
$$4 \times \boxed{r}$$

$$\frac{u}{\times 4}$$

factor	4	4	C	e	4	4	4	n		1
factor	r	S	4	4	e	v	$\boldsymbol{x}$	4	4	2
product	16	20	4	8	12	24	36	12	28	32

# Multiplication and division with four

1	2	3	4	5	6	7	8	9
	· ^	1	77	Lines.		Z	•	8
2=1	4	. La	大	7		1		×
		A	*	V	4	Ø	C	*
	•	<b>A</b>	*	-	<b>\rightarrow</b>		C	•
4	8	12	16	20	24	28	32	36

### EXERCISES

Write the numerals to complete the sentences.

- 1. 2 sets of 4 form 1 set of  $\overline{a}$ .
- 2. A set of 16 forms Z sets of 4.
- 3. 5 sets of 4 form 1 set of r.
- 4. A set of 20 forms e sets of 4.
- **5.** 7 sets of 4 form a set of  $\overline{n}$ .
- 6. 9 sets of 4 form a set of  $\overline{s}$ .
- 7. A set of 36 forms  $\overline{\boldsymbol{u}}$  sets of 4.
- 8. A set of 4 forms 4 sets of O.
- 9. 4 sets of  $\boxed{c}$  form a set of 12.
- **10.** 4 sets of **v** form a set of 24.
- 11. A set of 24 forms 4 sets of x.
- 12. A set of 32 forms 4 sets of  $\overline{z}$ .
- **13.** 4 sets of **s** form a set of 36.

Use the pattern at the top of the page to solve the equations.

14. 
$$a \times 4 = 12$$
  $4 \times c = 12$   $12 \div 4 = e$   $12 \div m = 4$ 

15. 
$$o \times 4 = 16$$
  $4 \times r = 16$   $16 \div 4 = s$   $16 \div w = 4$ 

16. 
$$v \times 4 = 24$$
  $4 \times x = 24$   $24 \div 4 = z$   $24 \div a = 4$ 

17. 
$$m \times 4 = 28$$
  $4 \times w = 28$   $28 \div 4 = \overline{w}$   $28 \div \overline{z} = 4$ 

17. 
$$m \times 4 = 28$$
  $4 \times m = 26$   $26 \div 4 = a$   $36 \div c = 4$ 

19. 
$$u \times 4 = 20$$
  $4 \times s = 20$   $20 \div 4 = s$   $20 \div z = 4$ 

# Using sets and numbers

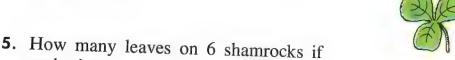
1. How many legs do 3 cows have? 5 cows? 2 cows? 9 cows?



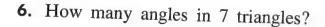
- 2. Three sets of four make one set of
- 3. How many legs for 4 birds? 6 birds? 8 birds? 7 birds?



4. How many sets of three are there in one set of 15?



each shamrock has 3 leaves?



7. If each window has 4 panes, how many panes are there in 6 windows?



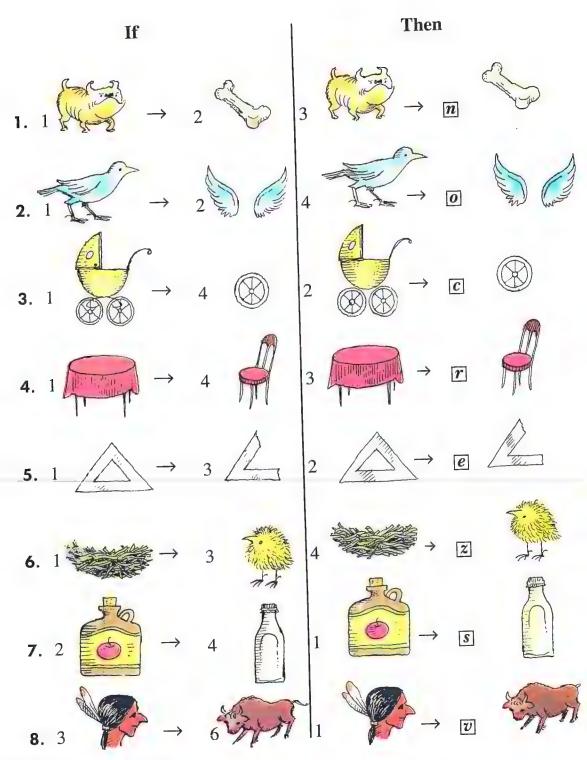
Just For Fun

8. To find the number of cows in the field, Bill counted the cows' legs and divided by 4. Is this a sensible way to find the number of cows? How many legs were there in the field? Don't forget to count Bill's.



# How many in each set?

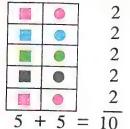
Complete the matchings.



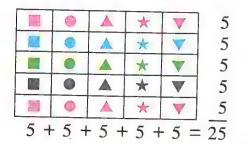
### Five as a factor



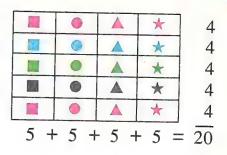
Use the patterns to solve the equations.



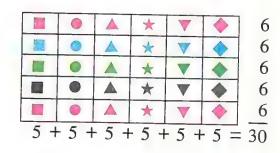
1. 
$$2 + 2 + 2 + 2 + 2 = \mathbf{a}$$
  
 $5 \times \mathbf{c} = 10$   
 $5 + 5 = \mathbf{c}$   
 $2 \times \mathbf{n} = 10$ 



3. 
$$5 \times 5 = x$$
  
 $5 \times z = 25$ 



2. 
$$4 + 4 + 4 + 4 + 4 = m$$
  
 $5 \times 0 = 20$   
 $5 + 5 + 5 + 5 = r$   
 $4 \times s = 20$ 



4. 
$$6 \times 5 = 0$$
  
 $5 \times 6 = 5$ 

- 1 this pattern we see that 6 sets of 5 may be joined
- o form 1 set of 30. So 7 sets of 5 form 1 set of 35.

-							001	0. 00	
	1	2	3	4	5	6	7	8	9
				*			8	C	8
				*	V		8		8
				*		<b>\</b>	8	C	8
				*		•		C	
				*			Ø	C	8
	5	10	15	20	25	30	35	40	45

ie the pattern to solve the equations.

5. 
$$7 \times 5 = a$$
  $5 \times 7 = c$ 

$$5 \times 7 = c$$

$$5 \times \boxed{e} = 35$$

$$7 \times \mathbf{n} = 35$$

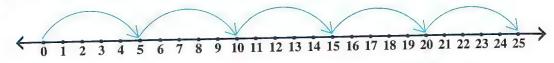
6. 
$$8 \times 5 = 0$$

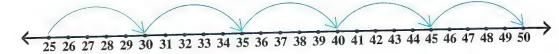
$$5 \times 8 = \mathbf{r}$$

$$5 \times \boxed{s} = 40$$

$$8 \times \mathbf{u} = 40$$

### Five as a factor





### EXERCISES

Use the number line above to complete this set:

Use the number line to rename these products.

**2.** 
$$2 \times 5$$

3. 
$$3 \times 5$$

**4.** 
$$4 \times 5$$

**6.** 
$$6 \times 5$$

Name the number on the number line just after:

10. 
$$2 \times 5$$
 11.  $6 \times 5$  12.  $5 \times 5$ 

12. 
$$5 \times 5$$

13. 
$$7 \times 5$$

Name the number on the number line just before:

14. 
$$9 \times 5$$

**14.** 
$$9 \times 5$$
 **15.**  $3 \times 5$  **16.**  $4 \times 5$ 

16. 
$$4 \times 5$$

Name the numbers on the number line between:

18. 
$$2 \times 5$$
 and  $3 \times 5$ 

19. 
$$5 \times 5$$
 and  $6 \times 5$ 

**20.** 
$$4 \times 5$$
 and  $3 \times 5$ 

**21.** 
$$4 \times 5$$
 and  $5 \times 5$ 

**22.** 
$$8 \times 5$$
 and  $9 \times 5$ 

**23.** 
$$6 \times 5$$
 and  $7 \times 5$ 

Name the products.

Name the missing factors.

$$\frac{\times r}{15}$$

$$\times 5$$

$$\frac{\times e}{25}$$

**2** 36. 
$$\times$$

$$\frac{5}{n}$$

$$\frac{\lambda}{20}$$
  $\frac{\lambda}{2}$ 

$$\frac{\times 5}{45}$$

$$\frac{\times n}{5}$$

$$\frac{\times x}{40}$$

factor	$\boldsymbol{x}$	5	C	n	5	5	5	r	S	5
factor	5	и	5	5	e	v	z	5	5	a
product	20	25	5	10	40	30	45	15	35	20

# Multiplication and division with five



1	2	3	4	5	6	7	8	9
			*	W	<b>\_</b>	8	C	8
			*	~	-	8	C	8
			*	-	•	8	C	8
	0	A	*			8	C	8
			*			8	C	8
5	10	15	20	25	30	35	40	45

### EXERCISES

Write the numerals to complete the sentences.

- 1. 2 sets of 5 form 1 set of a.
- 2. 4 sets of 5 form 1 set of [e].
- 3. A set of 20 forms  $\overline{r}$  sets of 5.
- **4.** 5 sets of 5 form 1 set of **C**.
- 5. A set of 35 forms v sets of 5.
- **6.** 5 sets of 3 form 1 set of  $\overline{\boldsymbol{u}}$ .
- 7. A set of 15 forms 5 sets of  $\mathbb{Z}$ .
- 8. 5 sets of 8 form 1 set of  $\mathbf{O}$ .
- 9. A set of 45 forms 5 sets of  $\overline{n}$ .

Jse the pattern at the top of the page to solve the equations.

10. 
$$a \times 5 = 15$$
  $5 \times s = 15$   $15 \div 5 = m$   $15 \div v = 5$ 

11. 
$$\mathbf{e} \times 5 = 20$$
  $5 \times \mathbf{c} = 20$   $20 \div 5 = \mathbf{u}$   $20 \div \mathbf{z} = 5$ 

12. 
$$\mathbf{r} \times 5 = 25$$
  $5 \times \mathbf{s} = 25$   $25 \div 5 = \mathbf{o}$   $25 \div \mathbf{x} = 5$ 

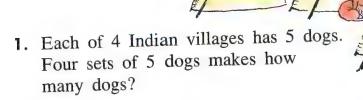
13. 
$$\vec{c} \times 5 = 30$$
  $5 \times \vec{a} = 30$   $30 \div 5 = \vec{e}$   $30 \div \vec{m} = 5$ 

14. 
$$\boxed{v} \times 5 = 35$$
  $5 \times \boxed{a} = 35$   $35 \div 5 = \boxed{c}$   $35 \div \boxed{e} = 5$ 

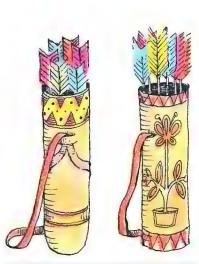
15. 
$$\mathbf{n} \times 5 = 40$$
  $5 \times \mathbf{u} = 40$   $40 \div 5 = \mathbf{r}$   $40 \div \mathbf{s} = 5$ 

**16.** 
$$\boxed{0} \times 5 = 45$$
  $5 \times \boxed{z} = 45$   $45 \div 5 = \boxed{u}$   $45 \div \boxed{v} = 5$ 

### Indians



- 2. There are 25 tepees in the Indian village. When the Indians move, the tepees go on sleds which hold 5 tepees each. How many sleds will be needed to hold all of the tepees?
- 3. There are 5 strings of 7 ponies. We know that 5 sets of 7 form 1 set of how many? There are how many ponies?
- 4. There are 9 feathers in each headdress. If there are 5 headdresses, how many feathers are there in all?
- **5.** Three Indian braves each have 5 arrows. How many arrows are there in all?
- **6.** Each quiver has 6 arrows in it. If there are 5 quivers, how many arrows are there in all?
- **7.** If each quiver holds 10 arrows, how many quivers are needed to hold 50 arrows?



# One and zero as factors



Solve these equations.

$$1. 1+1=\overline{a}$$

$$1 + 1 + 1 = [e]$$

3. 
$$1+1+1+1=\bar{x}$$

4. 
$$1 + 1 + 1 + 1 + 1 = \boxed{z}$$

$$2 \times 1 = \mathbf{u}$$

$$3 \times 1 = \boxed{r}$$

$$4 \times 1 = s$$

$$5 \times 1 = \boxed{c}$$

$$1 \times 2 = \overline{v}$$

$$1 \times 3 = x$$

$$1 \times 4 = \mathbf{m}$$
$$1 \times 5 = \mathbf{e}$$

Solve these equations.

2.

6.

5. 
$$0 + 0 = \bar{a}$$

$$0+0+0=\overline{n}$$

7. 
$$0 + 0 + 0 + 0 = 3$$

8. 
$$0 + 0 + 0 + 0 + 0 = x$$

$$2 \times 0 = c$$

$$3 \times 0 = x$$

$$4 \times 0 = \overline{u}$$

$$5 \times 0 = a$$

$$0 \times 2 = \boxed{e}$$

$$0 \times 3 = \mathbf{r}$$

$$0 \times 4 = \boxed{v}$$

$$0 \times 5 = \mathbf{z}$$

Solve these equations.

**9.** 
$$1 \times 4 = u$$

$$4 \times 1 = \mathbf{x} \qquad 4 \div 4 = \mathbf{a}$$

$$4 \div 4 = 0$$

$$4 \div 1 = |r|$$

10. 
$$6 \times 1 = e$$

$$1 \times 6 = n$$

$$6 \div 6 = \mathbf{c}$$

$$6 \div 1 = s$$

11. 
$$1 \times 7 = r$$

$$7 \times 1 = s$$

$$7 \div 7 = e$$

$$7 \div 1 = \mathbf{u}$$

12. 
$$8 \times 1 = \bar{a}$$

$$1 \times 8 = \mathbf{c}$$

$$8 \div 8 = n$$

$$8 \div 1 = v$$

13. 
$$4 \times 0 = n$$

$$0 \times 4 = \mathbf{u}$$

$$0 \div 4 = \boxed{z}$$

$$0 \div 1 = [x]$$

14. 
$$0 \times 3 = a$$

$$3 \times 0 = \boxed{e}$$

$$0 \div 3 = [r]$$

$$0 \div 5 = \boxed{z}$$

Vrite the words to complete the sentences.

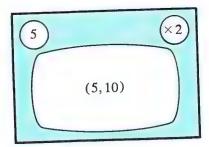
- 15. When 1 is one factor, the product is the same as the other \_\_?
- 16. When the product and one factor are the same, the missing factor is ?...
- 17. Given the product and one factor of 1, the missing factor is the same as the ?.
- 18. When zero is one factor, the product is \_\_?\_.
- 19. If there are no letters in your mailbox, and you look in your box 20 times, you will find ? letters.



# Number pairs and number patterns

Our Number Pair TV can use multiplication rules.

Turn the number knob to 5 and tune in the rule **multiply by 2.**  $5 \times 2 = 10$ . The screen shows the number pair (5, 10).



### EXERCISES

Write a multiplication rule to complete these sets of number pairs.

1. 
$$\{(2,4), (3,6), (4, @), (5, r), (6, C), (7, x), (9, e)\}$$

**2.** 
$$\{(2, 6), (3, 9), (5, \mathbb{C}), (7, \mathbb{X}), (6, \mathbb{n}), (8, \mathbb{e}), (9, \mathbb{s})\}$$

3. 
$$\{(2, 8), (3, 12), (4, \mathbb{Z}), (1, \mathbb{A}), (7, \mathbb{X}), (8, \mathbb{N}), (5, \mathbb{F})\}$$

**4.** 
$$\{(1, 5), (2, 10), (3, \boxed{e}), (6, \boxed{v}), (4, \boxed{r}), (9, \boxed{s}), (8, \boxed{c})\}$$

**5.** 
$$\{(1, 1), (2, 2), (3, 3), (4, 4), (5,  $\overline{u}$ ),  $(6, \overline{v}), (9, \overline{n})\}$$$

**6.** 
$$\{(1, 0), (2, 0), (3, 0), (4, 0), (5, \mathbb{Z}), (6, \mathbb{S}), (8, \mathbb{Z})\}$$

Write the numerals to complete these patterns.

8.	×	5	4	6	1	2	8	3	9
	3	15	12	e	u	a	x	r	n

Copy and complete these charts.

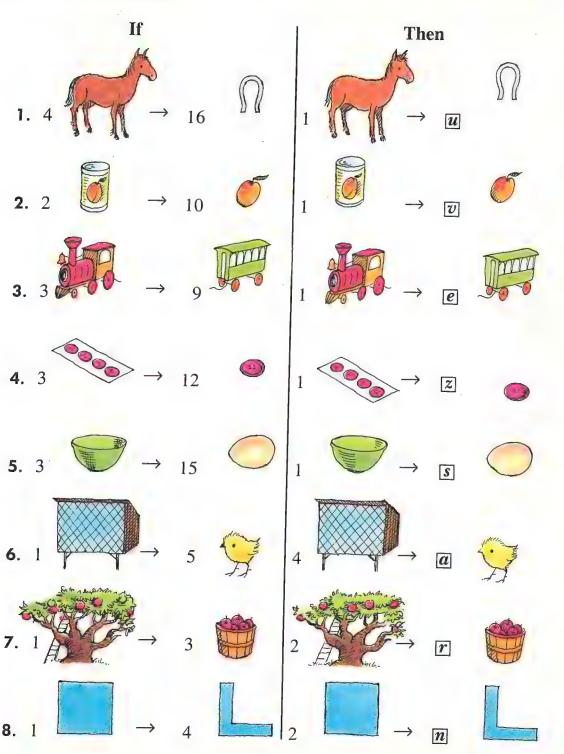
11.	×	1	2	3	4	5
	2	2	4	6	8	10
	3	3	6			
	4					
	3 4 5 6					
	6				L	
	7					
	8					
	9				L	

				_				
12.	1,2,	4,5, 6	7,8, 9		1 1/47	: ^\\.	1 TO 18	25,26, 27
	1,2, 3,4	5,6, 7,8					 . 79,5 	33,34, 35,36
	1,2, 3,4, 5	6,7, 8,9, 10						41,42, 43,44, 45

# How many in each set?

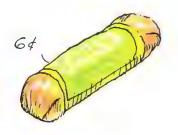


# Complete the matchings.



# How much will they cost?

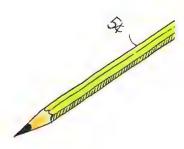
Write the numeral to replace each  $\square$ .



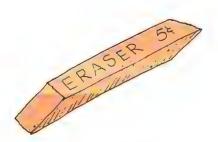
1. 4 cost □¢



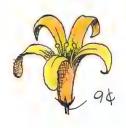
**2.** 3 cost □¢



**3.** 6 cost □¢



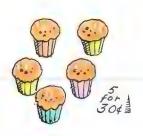
**4.** 8 cost □¢



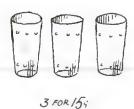
**5.** 5 cost □¢



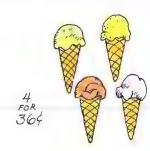
**6.** 9 cost □¢



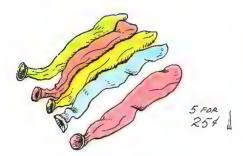
**7.** 1 costs □¢



**8.** 1 costs □¢



**9.** 1 costs □¢



**10.** 1 costs □¢



11. 1 costs □¢



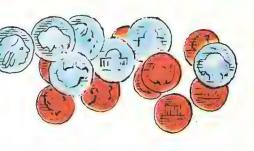
**12.** 1 costs □¢

148 (one hundred forty-eight)

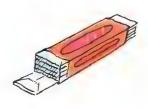
### Using sets and numbers



- 1. How many cents are there in 7 nickels?
- **2.** A set of 20 may be separated into how many sets of 4?
- **3.** A frog jumped a distance of 18 feet in 3 equal leaps. How long was each leap?
- **4.** Four sets of 6 make one set of  $\overline{n}$ .
- 5. If you had 20 baseball cards and wanted to put them into piles of 4, how many piles would you have?
- **6.** Four girls each had 5 dolls. How many dolls did they have together?
- **7.** How many 5 foot ropes can you cut from a 20 foot rope?
- **8.** If each team has 9 members, how many members are there on 4 teams?
- **9.** If each package of gum costs 4 cents, how much will it cost for 7 packages of gum?
- **0.** A set of 32 may be separated into 4 sets of r.







# Telling the time



The game begins at quarter past two.



The plane leaves at quarter to four.

### EXERCISES

Tell the time by writing past or to.



1. Half ? one



2. Half ?\_\_ ten



3. Half \_\_?\_\_ four







4. Quarter ? two 5. Quarter ? four 6. Quarter ? seven







7. Quarter ? two 8. Quarter ? four 9. Quarter ? nine

## Telling time



On a clock there are 5 minutes between each two numerals. Each numeral shows the number of sets of five. For example, the 1 shows 5 minutes because 1 set of 5 is 5. The 6 shows 30 minutes because 6 sets of 5 is 30.



The time shown on the clock is 35 minutes past two. We may write this as 2:35 or two thirty-five

### EXERCISES

Write the numerals to replace each □.





1. a minutes past r





2. *n* minutes past *e* 3. *x* minutes past *u* 4: [z]



4. r minutes past u 5. r minutes past r 6. r minutes past r1: e

9: r

 $10: \boldsymbol{r}$ 

ve minutes past two may be written as 2:05. Write her names for these:

- 7. five past three
- **8.** twenty past four
- 9. twenty-five past six

- 10. 3:25
- 11. 4:20
- 12. 7:05
- **13.** 9:00

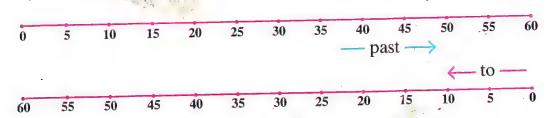
uarter past two may be written as 2:15. Write other imes for these:

- **14.** quarter past six
  - 15. quarter past ten
- 16. half past five

- 17. half past eight 18. fifteen past nine
- 19. ten past seven

### Telling time

A clock face is a kind of number line with points, from 0 to 60.



Instead of saying "50 minutes **past**" we can say "10 minutes **to**". Fifty minutes past 5 (5:50) may be written "10 minutes to 6."

### EXERCISES

Write the numerals to complete this set.

1. {60, 55, 50, 45, \overline{a}, \overline{c}, \overline{e}, \overline{n}, \overline{r}, \overline{s}, \overline{u}, 5, 0}

Write the numerals to replace each  $\square$ .







2.  $\overline{u}$  minutes to  $\overline{c}$  2:  $\overline{a}$ 



3. *e* minutes to *r* 7: *e* 



4.  $\overline{v}$  minutes to  $\overline{s}$  3: $\overline{n}$ 



5. v minutes past x

z minutes to n 8: e

6. a minutes past e

r minutes to s

5: X

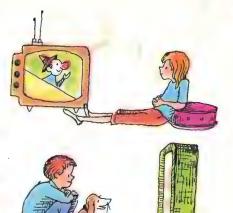
7.  $\overline{u}$  minutes past  $\overline{v}$   $\overline{z}$  minutes to  $\overline{x}$   $10: \overline{c}$ 

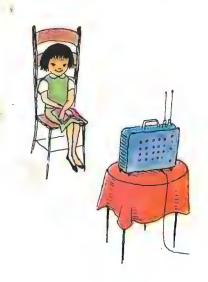
- 8. Write another name for ten minutes to eight.
- 9. Write another name for ten minutes past eight.

152 (one hundred fifty-two)

### Watching television

- 1. Nancy's favorite TV show begins at quarter past four and ends at half past four. How long is Nancy's favorite program?
- 2. Philip watches TV every day from five minutes to four until quarter past four.
  - write another name for five minutes to four and for quarter past four.
  - b. How many minutes does Philip water TV?
- 3. Joanne watches TV every day from three thirty-five to four o'clock.
  - **a.** Write another name for three thirty-five.
  - **b.** How many minutes does Joanne watch TV?
- 4. Betsy's two favorite TV cartoons are on Friday, one at quarter past three and the other at 3:15. Can she watch them both?
- 5. Tommy's favorite program begins at quarter to six. It lasts for 20 minutes. At what time does it end?
- School ends at half past three. It takes Harry 20 minutes to get home. The space program begins at a quarter to four.
  - **a.** Does Harry get home in time to see the beginning of the program?
  - b. How many minutes late is he?





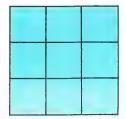




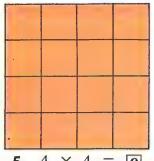
How many small squares?



1.  $2 \times 2 = a$ 

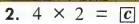


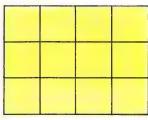
3.  $3 \times 3 = e$ 



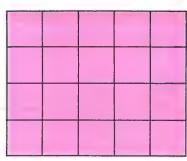
5. 
$$4 \times 4 = 0$$







4.  $4 \times 3 = n$ 

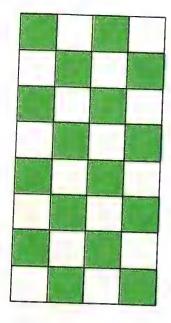


**6.** 
$$5 \times 4 = r$$

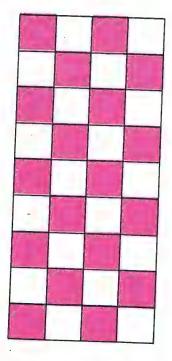


7.  $3 \times 9 = s$ 

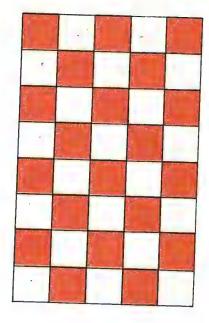




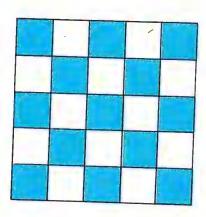
1. How many tiles?



3. How many tiles?



2. How many tiles?



4. How many tiles?

# CHECKING UP

Solve the equations.

1. 
$$3 \times 4 = a$$

4. 
$$3 \times e = 24$$

7. 
$$2 \times z = 12$$

10. 
$$4 \times a = 16$$

2. 
$$3 \times 9 = r$$

5. 
$$5 \times \bar{u} = 30$$

8. 
$$4 \times [r] = 20$$

11. 
$$n \times 5 = 45$$

3. 
$$4 \times 6 = C$$

**6.** 
$$6 \times v = 30$$

9. 
$$s \times 3 = 18$$

12. 
$$2 \times 9 = r$$

Name the products or missing factors.

27. 
$$\frac{\overline{u}}{\times 7}$$
 28.  $\frac{\overline{r}}{\times 9}$  29.  $\frac{5}{\times 5}$  30.  $\frac{\overline{c}}{\times 3}$  31.  $\frac{7}{\times 5}$  32.  $\frac{\overline{v}}{\times 2}$  33.  $\frac{9}{\times 5}$   $\frac{\times 7}{21}$  27  $\frac{\times 9}{27}$   $\frac{\times 5}{\overline{s}}$   $\frac{9}{9}$   $\frac{\times 5}{\overline{u}}$   $\frac{10}{10}$   $\frac{\times 5}{\overline{e}}$ 

Complete the sentences.

- 34. To multiply, you rename two ? as a product.
- **35.** To divide, you name a missing ?

Solve the equations.

**36.** 
$$14 \div 2 = \overline{a}$$
 **37.**  $15 \div 3 = \overline{n}$ 

**39.** 
$$30 \div 6 = e$$

**42.** 
$$x \div 2 = 4$$

**45.** 
$$s \div 3 = 4$$

37 
$$15 \div 3 = [n]$$

**40.** 
$$24 \div 4 = \overline{v}$$

**43.** 
$$a \div 3 = 6$$

**46.** 
$$\bar{n} \div 2 = 8$$

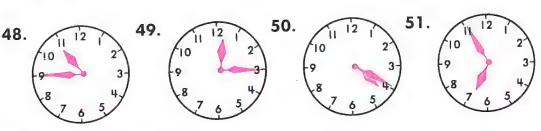
**38.** 
$$16 \div 2 = \boxed{c}$$

41. 
$$35 \div 5 = \mathbf{u}$$

**44.** 
$$\mathbf{z} \div 3 = 1$$

47.  $[r] \div 5 = 5$ 

Name the time shown on these clocks.



## LOOKING BACK



When you multiply, you begin with two factors and name a product.

Name the products when the factors are:

1. 6 and 3

2. 7 and 2

**3.** 5 and 4

**4.** 3 and 5

We have learned about 2, 3, 4, and 5 as factors.

### Complete the sets.

- 5. Write the products that have 2 as a factor,  $\{0, 2, 4, \overline{a}, \overline{e}, \overline{c}, \overline{n}, \overline{x}, \overline{r}, 18\}$
- **6.** Write the products that have 3 as a factor,  $\{0, 3, 6, \overline{u}, \overline{v}, \overline{s}, \overline{a}, \overline{z}, \overline{e}, 27\}$
- 7. Write the products that have 4 as a factor,  $\{0, 4, 8, \overline{a}, \overline{e}, \overline{c}, \overline{n}, \overline{z}, \overline{x}, 36\}$
- 8. Write the products that have 5 as a factor,  $\{0, 5, 10, \overline{u}, \overline{v}, \overline{c}, \overline{x}, \overline{z}, \overline{r}, 45\}$

To divide, you begin with a product and a factor, and name the other factor.

### hat is the other factor if

- 9. the product is 12 and one factor is 4?
- 10. the product is 18 and one factor is 3?
- 11. the product is 10 and one factor is 5?
- 12. the product is 27 and one factor is 3?

### ve the equations.

3. 
$$4 \times 7 = [a]$$

14. 
$$5 \times 6 = n$$

15. 
$$6 \times 4 = e$$

**6.** 
$$4 \times \mathbf{r} = 16$$

17. 
$$4 \times C = 8$$

18. 
$$3 \times \mathbf{z} = 21$$

9. 
$$12 \div 2 = s$$

**20.** 
$$12 \div 3 = u$$

**21.** 
$$35 \div 7 = v$$

**2.** 
$$a \div 3 = 2$$

**23.** 
$$u \div 3 = 8$$

**24.** 
$$r \div 5 = 9$$

# LOOKING WAY BACK

Name the sums.

me the sums.

1. 
$$34$$
 2.  $34$  3.  $34$  4.  $46$  5.  $30$  6.  $61$   $+21$   $+26$   $+26$   $+20$   $+40$   $+28$ 

Name the missing addends.

Write the numerals to complete each sentence.

- 31. The hundreds' digit in 763 is a.
- **32.** In the subtraction 9 6 = 3, the addends are o and r.

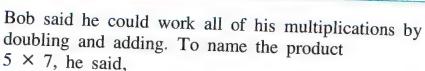
Name the differences.

**33.** 
$$37 - 4$$
 **34.**  $37 - 7$  **35.**  $34 - 7$ 

- 41. Name the number between 46 and 48 on the number line.
- 42. Name the number just after 956 on the number line.

158 (one hundred fifty-eight)

# EXTRA FOR EXPERTS



$$7 + 7 = 14$$
, that is,  $2 \times 7$   
 $14 + 14 = 28$ , that is,  $4 \times 7$   
 $28 + 7 = 35$ , that is,  $5 \times 7$ 

Use this doubling method to name these products.

1. 
$$5 \times 6$$

**2.** 
$$4 \times 6$$

**4.** 
$$5 \times 8$$

Solve the equations.

5. Since 
$$4 \times 3 = 12$$
 and  $4 \times 2 = 8$ ,  $4 \times 5 = \boxed{v}$ 

**6.** Since 
$$3 \times 2 = 6$$
 and  $3 \times 8 = 24$ ,  $3 \times \boxed{c} = \boxed{n}$ 

7. Since 
$$3 \times 5 = 15$$
 and  $3 \times 4 = 12$ ,  $3 \times @ = x$ 

Just For Fun

- 1. Play the Factor-Product Game with your neighbor. One of you says: "I'm thinking of two factors. They are 3 and 4." The other one says, "The product is 12." Since he is right, he takes a turn. He may say, "I'm thinking of a product and a factor. The product is 18 and the factor is 3." Then you should say, "The other factor is 6."
- 2. Make a list of times when you may need to multiply or divide. Your list may include some of these: finding the cost of a number of things which cost the same, sharing objects such as apples, or finding how many pairs of socks in a bundle.
- 3. Make up puzzles like these:
  - a. Name the number which gives a product of 6 when multiplied by 2.
  - **b.** Name a number which is between 5 and 10 and is a product with 4 as one factor.

# CHAPTER 6 · Fractions





1 of the 3 balls is red.

 $\frac{1}{3}$  of the balls is red.

One third of the balls is red.

2 of the 3 balls are green.

 $\frac{2}{3}$  of the balls are green.

Two thirds of the balls are green.



1 of the 4 balls is red.

 $\frac{1}{4}$  of the balls is red.

One fourth of the balls is red.

3 of the 4 balls are green.

 $\frac{3}{4}$  of the balls are green.

Three fourths of the balls are green.



1 of the 5 balls is red.

 $\frac{1}{5}$  of the balls is red.

One fifth of the balls is red.

4 of the 5 balls are green.

 $\frac{4}{5}$  of the balls are green.

Four fifths of the balls are green.

 $\frac{1}{3}$ ,  $\frac{2}{3}$ ,  $\frac{1}{4}$ ,  $\frac{3}{4}$ ,  $\frac{1}{5}$ ,  $\frac{4}{5}$ , are fractions. They are names for fractional numbers.

### EXERCISES

Write the fraction to compare the number of red balls with the total number of balls in each of these sets.













7-12. Write the fraction to compare the number of green balls with the total number of balls in each of the sets in questions 1 to 6.

Rename these fractions using digits:

13. one half

14. one third

15. one fourth

160 (one hundred sixty)

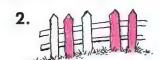
### **Fractions**



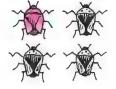
/rite fractions for the subsets that are red.

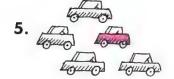












**7**.







10.



11.



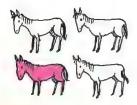
12.







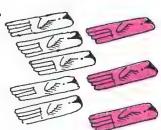




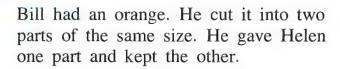
17.



18.



### Meaning of fractions



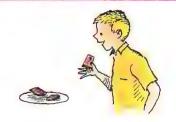




Bill has 1 part of the orange. There are 2 parts in all. Bill has **one half** of the orange. He gave Helen one half of the orange.

### EXERCISES

Bob had a candy bar. He cut it into three parts of the same size. He took one part and left the other two parts on the plate.



- 1. How many parts of the candy bar does Bob have?
- 2. How many parts are there in all?
- **3.** Complete this sentence: Bob has \_\_? \_\_ ? \_\_ of the candy bar.
- **4.** Complete this sentence: There are \_\_? \_\_? \_\_ of the candy bar on the plate.

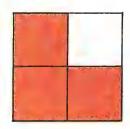
Jane had an apple. She cut it into four parts, each the same size. She took one part and left the other three on the plate.



- 5. How many parts of the apple does Jane have?
- 6. How many parts are there in all?
- **7.** Complete this sentence: Jane has one \_\_\_?\_\_ of the apple.
- 8. How many of the parts are on the plate?
- **9.** Complete this sentence: \_\_? \_\_? of the apple are on the plate.

# Matching regions





In this square, 3 of the 4 parts are shaded. The fraction for the shaded region is  $\frac{3}{4}$ .

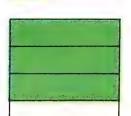
### EXERCISES

Vrite the fraction for  ${\bf a}$ . the shaded region  ${\bf b}$ . the shaded region.

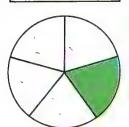
1.



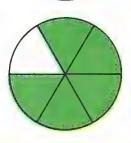
4.



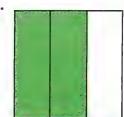
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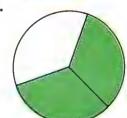
10.



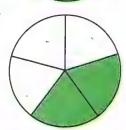
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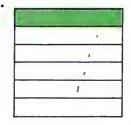
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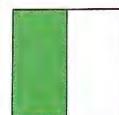
8.



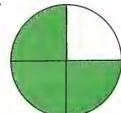
11.



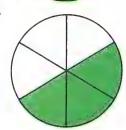
3.



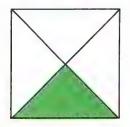
6.



9.



12.



The rod has 2 parts of the same length; 1 of the 2 parts is shaded. The fraction for the shaded part is $\frac{1}{2}$ . One half of the rod is shaded.
The rod has 3 parts of the same length; 2 of the 3 parts are shaded. The fraction for the shaded part is $\frac{2}{3}$ . Two thirds of the rod is shaded.

## EXERCISES

Write the fraction for the part of each rod that is green.

1.	2.	
3.	4.	
5.	6.	
7.	8.	
9.	10.	
11.	12.	
13.	14.	
15.	16.	
1 <i>7</i> .	18.	

### Number line





The number line is shown in 2 parts of the same length. Y labels 1, so M labels  $\frac{1}{2}$ .



The number line is shown in 4 parts of the same length. Y labels 1, so R labels  $\frac{1}{4}$  and S labels  $\frac{3}{4}$ .

### EXERCISES

Name the fractional number labeled by M on each of these number lines.

1. 
$$\langle X \qquad M \qquad Y \rangle$$

5. 
$$\langle \frac{X}{0}, \frac{M}{1} \rangle$$

7. 
$$\langle X \rangle M \rangle Y$$

9. 
$$\langle X M \rangle$$

11. 
$$\langle \frac{X}{0} \rangle$$

13. 
$$\langle \frac{X \quad M}{0} \rangle$$

15. 
$$\langle \frac{X}{0}, \frac{M}{1}, \frac{Y}{1} \rangle$$

2. 
$$\langle X \rangle$$
  $M Y \rangle$ 

6. 
$$\langle \frac{X}{0} \quad M \quad Y \rangle$$

8. 
$$\stackrel{X}{\stackrel{M}{\rightleftharpoons}}$$

10. 
$$\langle \frac{X}{0}$$
  $M$   $Y$ 

12. 
$$\langle \frac{X}{0} \rangle$$

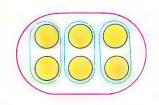
14. 
$$\langle \frac{X}{0}, \frac{M}{1} \rangle$$

16. 
$$\langle \frac{X}{0} \rangle$$

(one hundred sixty-five) 165

### Division and fractions

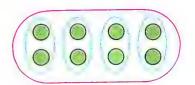
There are 3 sets of 2 in a set of 6.  $3 \times 2 = 6$ , so  $6 \div 3 = 2$ .  $\frac{1}{3}$  of 6 is 2.



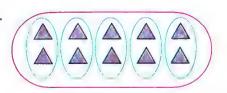
### EXERCISES

Complete these sentences.

- 1. There are  $\overline{a}$  sets of 2 in a set of 8.
- **2.**  $x \times 2 = 8$ , so  $8 \div c = 2$ .
- 3. [e] of 8 is 2.



- **4.** There are  $\overline{a}$  sets of 2 in a set of 10.
- 5.  $\mathbf{n} \times 2 = 10$ , so  $10 \div \mathbf{c} = 2$ .
- 6. v of 10 is 2.



- 7. There are  $\overline{a}$  sets of 3 in a set of 9, so 3 is  $\overline{u}$  of 9.
- 8. There are  $\overline{c}$  sets of 4 in a set of 12, so 4 is  $\overline{v}$  of 12.
- **9.** There are  $\mathbf{e}$  sets of 2 in a set of 10, so 2 is  $\mathbf{z}$  of 10.
- **10.** There are  $\overline{n}$  sets of 5 in a set of 20, so 5 is  $\overline{r}$  of 20.
- 11. There are  $\mathbf{z}$  sets of 4 in a set of 16, so 4 is  $\mathbf{x}$  of 16.
- 12. There are x sets of 6 in a set of 18, so 6 is  $\alpha$  of 18.

Solve these equations.

**13.** 
$$6 \div 3 = \mathbf{a}$$
,  $2 = \mathbf{n}$  of 6

15. 
$$8 \div 2 = [\overline{c}], 4 = [\overline{u}] \text{ of } 8$$

17. 
$$10 \div 2 = [e], 5 = [v] \text{ of } 10$$

**19.** 
$$12 \div 4 = x$$
,  $3 = 0$  of 12

**21.** 
$$15 \div 5 = [z], 3 = [x] \text{ of } 15$$

**14.** 
$$8 \div 4 = \overline{r}$$
,  $2 = \overline{c}$  of 8

**16.** 
$$9 \div 3 = S$$
,  $3 = X$  of  $9$ 

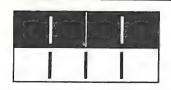
**18.** 
$$4 \div 2 = e$$
,  $2 = n$  of 4

**20.** 
$$12 \div 3 = \overline{a}$$
,  $4 = \overline{z}$  of  $12$ 

**22.** 
$$16 \div 4 = \overline{u}$$
,  $4 = \overline{r}$  of 16

### Fractions and rods





Two 2-rods match one 4-rod.  $2 \times 2 = 4$ , so  $4 \div 2 = 2$ .  $\frac{1}{2}$  of 4 is 2.

### EXERCISES

Use the rods to help you complete the equations.

1. 
$$2 \times \mathbf{a} = 6$$
  
 $\frac{1}{2}$  of 6 is  $\mathbf{e}$   
 $3 \times \mathbf{o} = 6$   
 $\frac{1}{3}$  of 6 is  $\mathbf{s}$ 

$$\begin{array}{ccc}
\hline{c} \times 2 &= 6 \\
6 \div 2 &= \boxed{n} \\
\hline{r} \times 3 &= 6
\end{array}$$

 $6 \div 3 = \overline{u}$ 



2. 
$$2 \times a = 8$$
  
 $\frac{1}{2}$  of 8 is  $e$   
 $4 \times o = 8$   
 $\frac{1}{4}$  of 8 is  $s$ 

$$\boxed{c} \times 2 = 8$$
  
 $8 \div 2 = \boxed{n}$ 

$$8 \div 2 = \mathbf{n}$$

$$\mathbf{r} \times 4 = 8$$

 $8 \div 4 = [v]$ 



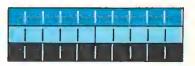
3. 
$$2 \times \mathbf{a} = 10$$
  
 $\frac{1}{2}$  of 10 is  $\mathbf{e}$   
 $5 \times \mathbf{o} = 10$   
 $\frac{1}{5}$  of 10 is  $\mathbf{s}$ 

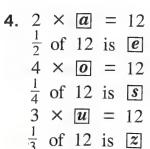
$$\mathbf{c} \times 2 = 10$$

$$10 \div 2 = \boxed{n}$$

$$r \times 5 = 10$$

$$10 \div 5 = \boxed{v}$$



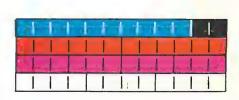


$$\begin{array}{cccc}
\hline{c} \times 2 &=& 12 \\
12 & \div 2 &=& \boxed{n}
\end{array}$$

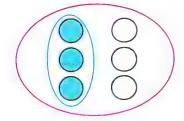
$$12 \div 4 = \boxed{v}$$

$$x \times 3 = 12$$

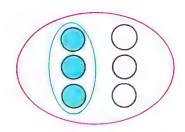
$$12 \div 3 = \mathbf{a}$$



### Fractions and sets



There are 2 subsets in the set. The blue subset is  $\frac{1}{2}$  of the set.



There are 6 objects in the set. 3 of the 6 objects are blue. The blue subset is  $\frac{3}{6}$  of the set.

 $\frac{3}{4}$  or  $\mathbf{e}$ 

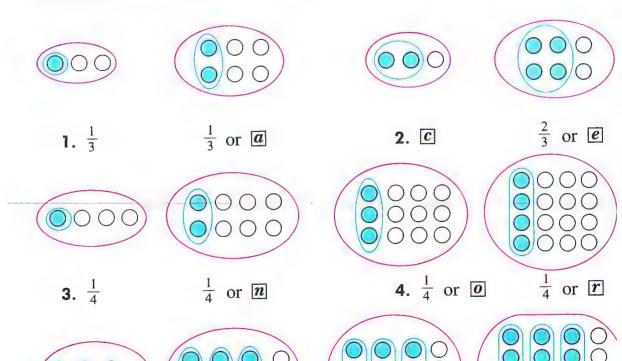
**6.**  $\frac{3}{4}$  or **u** 

 $\frac{1}{2}$  and  $\frac{3}{6}$  name the same fractional number.

### EXERCISES

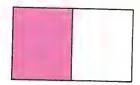
Name the fractional numbers shown by the blue subsets.

 $\frac{3}{4}$  or s



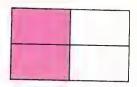
# Fractions and regions





The region shows 2 parts of the same size. The red region is  $\frac{1}{2}$  of the

whole region.

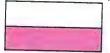


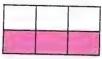
The region shows 4 parts of the same size. The red region is  $\frac{2}{4}$  of the whole region.

# $\frac{1}{2} = \frac{2}{4}$

### EXERCISES

Name the fractional numbers shown by the red regions.

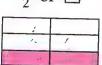








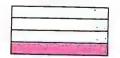






1. a







 $\boldsymbol{a}$ 

$$\frac{1}{4}$$
 or  $s$ 

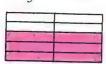




|z|7.

$$\frac{1}{5}$$
 or  $\mathbf{e}$ 





9. n

$$\frac{3}{5}$$
 or  $x$ 















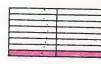


10. C









 $\frac{2}{5}$  or  $\mathbf{0}$ 

 $\frac{1}{2}$  or  $\boxed{0}$ 

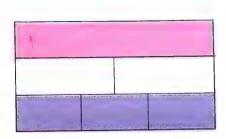
 $\frac{2}{3}$  or  $\mathbf{C}$ 

 $\frac{3}{4}$  or  $\overline{v}$ 

or v

(one hundred sixty-nine) 169

# Greater than—less than, with fractions

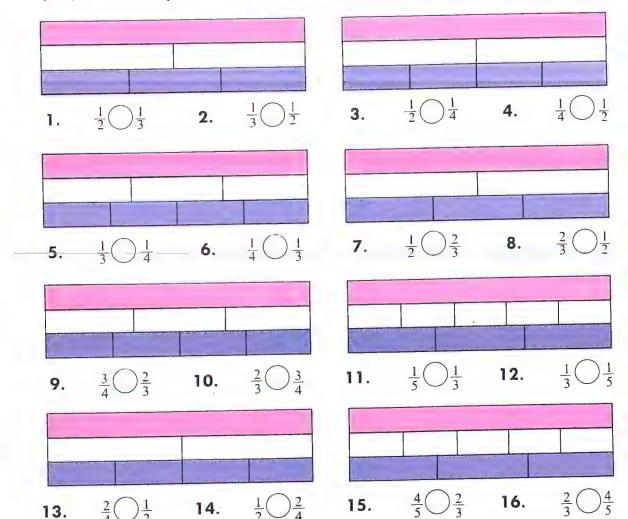


170 (one hundred seventy)

Each white rod is  $\frac{1}{2}$  of the red rod. Each purple rod is  $\frac{1}{3}$  of the red rod. The purple rods are shorter than the white rods, so  $\frac{1}{3}$  is less than  $\frac{1}{2}$ ,  $\frac{1}{3} < \frac{1}{2}$ . The white rods are longer than the purple rods, so  $\frac{1}{2}$  is greater than  $\frac{1}{3}$ ,  $\frac{1}{2} > \frac{1}{3}$ .

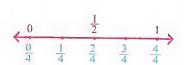
### EXERCISES

Study the rods and complete the sentences. Write <, >, or = to replace each  $\bigcirc$ .



### Greater than—less than





Fractional numbers to the left on the number line are less than numbers to the right, so  $\frac{1}{2} < \frac{3}{4}$ .

Fractional numbers to the right on the number line are greater than numbers to the left, so  $\frac{1}{2} > \frac{1}{4}$ .

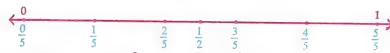
Numbers at the same place on the number line are equal, so  $\frac{2}{4} = \frac{1}{2}$ .

### EXERCISES

tudy these number lines and complete the number entences. Write <, >, or = to replace each  $\bigcirc$ .



- 1.  $\frac{1}{2}$  is to the right of  $\frac{1}{3}$ , so  $\frac{1}{2}$   $\bigcirc$   $\frac{1}{3}$ .
- **2.**  $\frac{1}{3}$  is to the right of  $\frac{1}{4}$ , so  $\frac{1}{3}$   $\bigcirc$   $\frac{1}{4}$ .
- **3.**  $\frac{1}{2}$  is to the left of  $\frac{2}{3}$ , so  $\frac{1}{2}$   $\bigcirc$   $\frac{2}{3}$ .
- **4.**  $\frac{3}{3}$  is at the same place as 1, so  $\frac{3}{3}$  1.

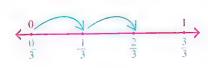


- **5.**  $\frac{1}{2}$  is to the right of  $\frac{2}{5}$ , so  $\frac{1}{2}$   $\bigcirc$   $\frac{2}{5}$ .
- **6.**  $\frac{1}{5}$  is to the left of  $\frac{2}{5}$ , so  $\frac{1}{5}$   $\bigcirc$   $\frac{2}{5}$ .
- 7.  $\frac{3}{5}$  is to the right of  $\frac{2}{5}$ , so  $\frac{3}{5}$   $\bigcirc$   $\frac{2}{5}$ .
- **8.**  $\frac{5}{5}$  is at the same place as 1, so  $\frac{5}{5}$  1.



- **9.**  $\frac{3}{6}$  is to the right of  $\frac{1}{6}$ , so  $\frac{3}{6}$   $\frac{1}{6}$ .
- 10.  $\frac{3}{6}$  is at the same place as  $\frac{1}{2}$ , so  $\frac{3}{6}$   $\bigcirc$   $\frac{1}{2}$ .

# Adding fractional numbers on the number line

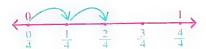


The number line shows 3 parts of the same length. Each move is  $\frac{1}{3}$ . So  $\frac{1}{3}$  and  $\frac{1}{3}$  is  $\frac{2}{3}$ .

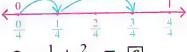
$$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

### EXERCISES

Study the number lines and complete the equations.

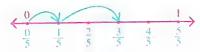


1. 
$$\frac{1}{4} + \frac{1}{4} = a$$

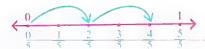


2. 
$$\frac{1}{4} + \frac{2}{4} = e$$

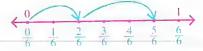
3. 
$$\frac{1}{5} + \frac{1}{5} = \boxed{c}$$



4. 
$$\frac{1}{5} + \frac{2}{5} = r$$



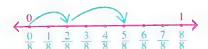
5. 
$$\frac{2}{5} + \frac{2}{5} = n$$



6. 
$$\frac{2}{6} + \frac{3}{6} = \bar{u}$$

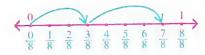


7. 
$$\frac{1}{6} + \frac{3}{6} = n$$



9. 
$$\frac{2}{8} + \frac{3}{8} = v$$

8. 
$$\frac{1}{6} + \frac{4}{6} = s$$



10. 
$$\frac{3}{8} + \frac{4}{8} = \mathbb{Z}$$

172 (one hundred seventy-two)

# Adding fractional numbers with rods





 $\frac{1}{2}$  and  $\frac{1}{2}$  is 1,  $\frac{1}{2} + \frac{1}{2} = 1$ ,  $\frac{1}{2}$  and  $\frac{1}{2}$  is  $\frac{2}{2}$ , so  $\frac{2}{2}$  is another name for 1.

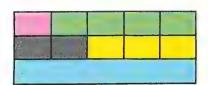
#### EXERCISES

Study the rods and complete the equations.



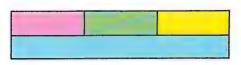
1. 
$$\frac{1}{3} + \frac{2}{3} = \frac{a}{3}$$
 or  $v$ 

$$\frac{2}{3} + \frac{1}{3} = \frac{x}{3}$$
 or **n**



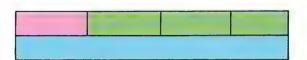
3. 
$$\frac{1}{5} + \frac{4}{5} = \frac{\vec{r}}{5}$$
 or  $\vec{x}$ 

$$\frac{2}{5} + \frac{3}{5} = \frac{\$}{5}$$
 or  $e$ 



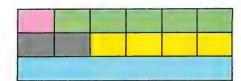
5. 
$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \mathbf{n}$$
 or  $\mathbf{x}$ 

$$\frac{1}{3} + \frac{1}{3} = \frac{z}{3}$$



2. 
$$\frac{1}{4} + \frac{3}{4} = \frac{C}{4}$$
 or  $U$ 

$$\frac{3}{4} + \frac{1}{4} = \frac{|r|}{4}$$
 or  $|e|$ 



4. 
$$\frac{1}{6} + \frac{5}{6} = \frac{\boxed{n}}{6}$$
 or  $\boxed{e}$ 

$$\frac{2}{6} + \frac{4}{6} = \frac{a}{6}$$
 or  $r$ 



**6.** 
$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{C}{4}$$
 or  $T$ 

$$\frac{1}{4} + \frac{1}{4} = \frac{\boxed{s}}{4}, \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{\boxed{u}}{4}$$

(one hundred seventy-three) 173

# Measuring

Measure these line segments to the nearest half inch.

1.

2.

3.

4.

Measure these line segments to the nearest fourth inch.

5.

6. .\_\_\_\_

7.

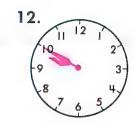
8.

What is the time shown on these clocks?

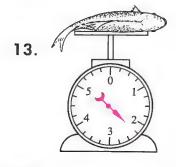


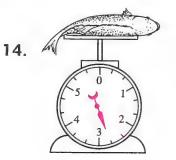


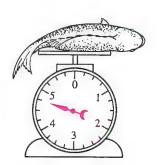




What is the weight shown on these scales?







174 (one hundred seventy-four)

(5)

Study these pictures and the chart.







2 pints→1 quart



4 quarts→1 gallon

										7 4	uarts	-ı gai	1011	
						gal	lon							
	art			q	uart			(	quart				quar	t
pint	pi			nt	pi	nt	pi	nt	pi	nt	pi	nt	-	
cup cup	cup	cup	cup	cup	cup	cup	cup	cup	cup	cup	cup.	cup	cup	CUD
								_ A	Ι.	T.P	Сар	cup	cup	cup

### EXERCISES

Vrite the numerals to complete these sentences.

- 1. 2 cups make 1 pint, so 1 cup makes a pint.
- 2. 2 pints make 1 quart, so 1 pint makes c quart.
- 3. 4 cups make 1 quart, so 1 cup makes e quart.
- **4.** 2 cups make **n** quart; 3 cups make **r** quart.
- **5.** 4 quarts make [r] gallon, so 1 quart makes [s] gallon.
- **6.** 2 quarts make **u** gallon; 3 quarts make **x** gallon.
- 7. 8 pints make x gallon, so 1 pint makes a gallon.
- 8. 2 pints make c gallon; 4 pints make e gallon.

lve these problems.

- 9. Milk is selling for 26¢ a quart. What is the cost of a. 2 quarts? b. 1 gallon?
- 10. Ice cream is selling for 29¢ a pint. What is the cost of a. a quart of ice cream?
  b. ½ gallon of ice cream?
- 1. Marlene is selling lemonade for 10¢ a cup. What does a quart of lemonade cost?



1. Irene and Rita are selling lemonade for 5¢ a cup.

a. How much will one pint of lemonade cost?

b. How much will one quart of lemonade cost?

- 2. Mr. Mays asked for <sup>1</sup>/<sub>2</sub> quart of lemonade. a. How many cups was this? b. How much should the girls charge him?
- 3. Between 11 o'clock and 12 o'clock the girls sold 10 cups of lemonade. a. How many pints was this?
  b. How many quarts?
- 4. Walt bought 6 cups of lemonade, and Betsy bought 1½ quarts. a. Did the children buy the same amount of lemonade? b. How much did Walt pay for his lemonade?
- 5. By noon Irene had sold 3 pints of lemonade and Rita 1 quart. a. Who sold more lemonade? b. How many cups more?
- **6.** One lady bought 2 pints of lemonade. **a.** What part of a gallon did she buy? **b.** How much did it cost?
- 7. At the end of the day the girls had sold 5 quarts of lemonade. a. How many gallons of lemonade was this? b. How much money did they make?



- 1. John mowed  $\frac{1}{2}$  of his yard in the morning and  $\frac{1}{3}$  of it in the afternoon. Did he mow more in the afternoon or in the morning?
- **2.** Kevin mowed  $\frac{1}{4}$  of his yard on Thursday and  $\frac{1}{3}$  of it on Friday. On which day did he mow more?
- 3. If Michael mowed  $\frac{1}{4}$  of his yard in the morning and  $\frac{3}{4}$  in the afternoon, would the entire yard be mowed?
- **4.** Lee mowed  $\frac{1}{3}$  of his yard before supper and  $\frac{1}{3}$  after supper. How much of the yard did he mow?
- 5. Hal mowed  $\frac{1}{4}$  of his yard in the morning and  $\frac{2}{4}$  in the afternoon. How much of the yard did he mow?
- **6.** Billie and Mike wanted to help Tom mow his lawn. Billie mowed  $\frac{1}{5}$ , Mike  $\frac{1}{5}$ , and Tom  $\frac{1}{5}$ . How much of the yard did they mow?
- 7. If Dad mowed  $\frac{4}{6}$  of the yard and Freddie mowed  $\frac{2}{6}$ , did they mow the whole yard?
- 3. Harry mowed  $\frac{2}{4}$  of the lawn before lunch. He mowed  $\frac{1}{4}$  of the lawn after lunch. How much lawn was not mowed?

# Number patterns

Write the numerals to complete these number patterns.

1. 
$$30 + 4 = 34$$
  
 $40 + 8 = +48$   
 $70 + 12 = 82$ 

Name these sums.

47

Write the numerals to complete these number patterns.

Name these missing addends.

# Number patterns



Write the numerals to complete these number patterns.

5.

1. 
$$300 + 20 + 6 = 326$$
  
 $400 + 50 + 3 = +453$   
 $700 + 70 + 9 = \boxed{a}$ 

600 30 9	$\overline{c}$
<b>4.</b> 300 40 8	$\overline{a}$

6.	300	20	7	u
	400	30	2	r
	200	30	9	x
	a	C	e	0

Name these sums.

Write the numerals to complete these number patterns.

lame the missing addends.



# Adding hundreds and tens

Study these methods for naming the sum 360 + 270.

Column Method	Three-step Method	Short Method
H T O 3 6 0 +2 7 0 5 13 0 6 3 0	$   \begin{array}{r}     360 \\     + 270 \\ \hline     \hline     130 & \textbf{Step 1:} 70 + 60 = 130 \\     500 & \textbf{Step 2:} 300 + 200 = 500 \\ \hline     \hline     630 & \textbf{Step 3:} 500 + 130 = 630   \end{array} $	$     \begin{array}{r}                                     $

#### EXERCISES

Name these sums using the column method.

Name these sums using the 3-step method.

Name these sums using the short method.



study these methods for naming the sum 362 + 276.

<b>Column Method</b>		Four-ste	ep Method	Short Method
H T O 3 6 2 +2 7 6 5 13 8 6 3 8	$ \begin{array}{r} 362 \\ +276 \\ \hline 8 \\ 130 \\ 500 \\ \hline 638 \end{array} $	Step 1: Step 2: Step 3: Step 4: 5	2 + 6 = 8 $ 60 + 70 = 130 $ $ 300 + 200 = 500 $ $ 500 + 130 + 8 = 638$	$   \begin{array}{r}     1 \\     362 \\     + 276 \\     \hline     638   \end{array} $

#### EXERCISES

ame these sums using the column method.

ame these sums using the 4-step method.

ame these sums using the short method.



Study these methods for naming the sum 357 + 289.

Column Method	Four-step Method	Short Method
H T O 3 5 7 +2 8 9 5 13 16 6 4 6	$   \begin{array}{r}     357 \\     +289 \\ \hline     16                               $	$\frac{11}{357} + 289 = 646$

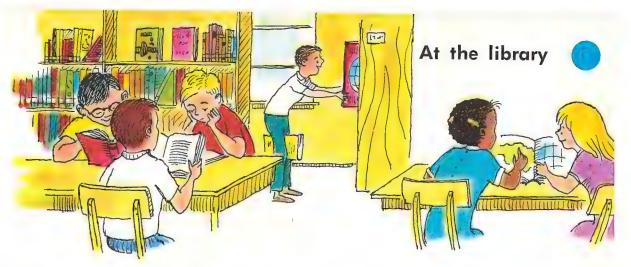
#### EXERCISES

Name these sums using the column method.

Name these sums using the 4-step method.

11. 
$$\frac{146}{+278}$$

Name these sums using the short method.



- 1. There are 120 chairs and 70 tables in the library. What is the total number of chairs and tables?
- 2. On one book shelf there are 350 books about children and 180 about animals. How many books are there about children and animals?
- 3. One hundred thirty-four children came to the library before 12:00. In the afternoon 149 children visited the library. How many children in all went to the library that day?
- I. The librarian told Elaine that 143 library books were taken out yesterday and 292 today. How many books were taken out on the two days?
- Linda read a total of 168 pages on Thursday. She read 535 pages during the next 2 days. How many pages did she read in the 3 days?
- . On Monday 483 children visited the library. On Tuesday 352 children visited the library. How many children went to the library on the two days?
- . There were 426 book shelves in the library, and 285 more were added. How many book shelves are there now?



Study these methods for naming the sum 420 + 950.

Column Method	Three-step Method	Short Method
Th H T O	$   \begin{array}{r}       420 \\       + 950 \\ \hline       70 \\       \hline       70 \\       \hline       1300 \\ \hline       \hline       1300 \\ \hline       \hline       1370 \\ \hline       \hline       3tep 1: 20 + 50 = 70 \\       \hline       400 + 900 = 1300 \\ \hline       \hline       1370 \\ \hline    $	420 + 950 1370

#### EXERCISES

Name these sums using the column method.

Name these sums using the 3-step method.

Name these sums using the short method.



Study these methods for naming the sum 450 + 970.

	Colui Meth			Three-step Method	Short Method
<u>Th</u> +	H 4 9 13 4	T 5 7 12 2	0 0 0 0	$   \begin{array}{r}     450 \\     + 970 \\ \hline     120 \text{ Step 1:} & 70 + 50 = 120 \\     1300 \text{ Step 2:} & 900 + 400 = 1300 \\ \hline     1420 \text{ Step 3:} & 1300 + 120 = 1420 \end{array} $	450 + 970 1420

#### EXERCISES

lame these sums using the column method.

ame these sums using the 3-step method.

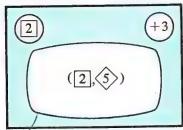
ame these sums using the short method.

(one hundred eighty-five) 185

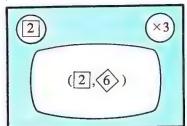


 $A = \{(2,5), (3,6), (4,7), (5,8), (6,9)\}$ 

 $B = \{(2,6),(3,9),(4,12),(5,15),(6,18)\}$ 



The tyned in rule to name the second number from the first number in set A is add 3. If the first number is shown as □ and the second number as ♦, the equation for the rule is  $\square + 3 = \lozenge$ .



The tuned in rule to name the second number from the first number in set B is multiply by 3. If the first number is shown as 
and the second number as  $\diamond$ , the equation for the rule is  $3 \times \square = \diamondsuit$ .

#### EXERCISES

Use the equations to complete the number pairs.

- 1.  $\Box + 4 = \diamond : \{(2, \mathbf{a}), (3, \mathbf{c}), (4, \mathbf{e}), (5, \mathbf{n}), (6, \mathbf{r})\}$
- **2.**  $\square \times 5 = \lozenge : \{(1, \mathbf{r}), (2, \mathbf{s}), (3, \mathbf{u}), (4, \mathbf{v}), (5, \mathbf{x})\}$
- **3.**  $\Box$  3 =  $\Diamond$ : {(9,  $\overline{z}$ ), (8,  $\overline{a}$ ), (7,  $\overline{c}$ ), (6,  $\overline{e}$ ), (5,  $\overline{n}$ )}
- **4.**  $\Box$  + 6 =  $\Diamond$ : {(1,  $\overline{a}$ ), (2,  $\overline{c}$ ), (6,  $\overline{n}$ ), (8,  $\overline{e}$ ), (3,  $\overline{v}$ )}
- **5.**  $\square \times 4 = \lozenge : \{(1, \mathbf{a}), (3, \mathbf{n}), (7, \mathbf{r}), (5, \mathbf{x}), (9, \mathbf{s})\}$

Write equations for the rules to complete these number pairs.

- **6.**  $\{(1,6), (2,7), (3,8), (4,9)\}$
- **7.** {(1, 2), (2, 4), (3, 6), (4, 8)}
- **8.**  $\{(8,6),(7,5),(6,4),(5,3)\}$
- **9.** {(10, 5), (8, 4), (6, 3), (4, 2)}
- **10.**  $\{(1,7), (2,8), (3,9), (4,10)\}$  **11.**  $\{(18,6), (15,5), (12,4), (9,3)\}$

Write the numerals to complete these number patterns.

#### CHECKING UP



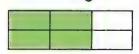
Solve these problems.

- 1. Write the fraction for the cones that are chocolate.
- **2.** Tommy shoveled  $\frac{1}{4}$  of the sidewalk before lunch. How much of the sidewalk is left to be shoveled?
- 3. Write the fraction for the window panes that still have to be cleaned.

Write other fractions for the shaded regions.

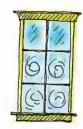






**5.** 
$$\frac{2}{3}$$
 or  $e$ 







**6.** 
$$\frac{3}{4}$$
 or **n**

solve these equations.

**7.** 
$$9 \div 3 = \mathbf{n}$$
, so  $3 = \mathbf{0}$  of  $9$  **8.**  $10 \div 2 = \mathbf{r}$ , so  $5 = \mathbf{s}$  of  $10$ 

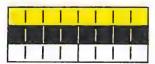
8. 
$$10 \div 2 = \mathbf{r}$$
, so  $5 = \mathbf{s}$  of 10

**9.** 
$$8 \div 2 = \overline{u}$$
, so  $4 = \overline{v}$  of  $8$  **10.**  $8 \div 4 = \overline{x}$ , so  $2 = \overline{z}$  of  $8$ 

**10.** 
$$8 \div 4 = \mathbf{z}$$
, so  $2 = \mathbf{z}$  of 8

itudy the rods and complete the sentences using <, >, or to replace each O.



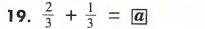


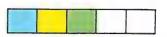


- $\frac{1}{2}\bigcirc\frac{1}{3}$  12.  $\frac{2}{3}\bigcirc\frac{1}{2}$  13.  $\frac{4}{5}\bigcirc\frac{2}{3}$  14.  $\frac{1}{3}\bigcirc\frac{2}{6}$   $\frac{2}{5}\bigcirc\frac{1}{3}$  16.  $\frac{4}{8}\bigcirc\frac{1}{2}$  17.  $\frac{1}{4}\bigcirc\frac{1}{2}$  18.  $\frac{3}{5}\bigcirc\frac{4}{5}$

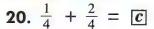
omplete these equations.

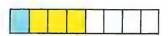




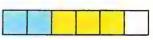


**22.** 
$$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \boxed{0}$$
 **23.**  $\frac{1}{8} + \frac{3}{8} = \boxed{r}$ 





23. 
$$\frac{1}{8} + \frac{3}{8} = r$$



21. 
$$\frac{2}{6} + \frac{3}{6} = n$$



**24.** 
$$\frac{1}{4} + \frac{3}{4} = s$$

# LOOKING BACK

We may write fractions for

- a. subsets of a given set,
- **b.** parts of a given region.

Name the fraction for the shaded subset.

1.





3.



4.



5.



Write the numerals to complete these sentences.

- **6.** There are **a** sets of 4 in a set of 8, so 4 is [e] of 8.
- 7. There are n sets of 5 in a set of 20, so 5 is z of 20.

Use the colored rods to help you complete the equations.

**8.** 
$$2 \times \bar{c} = 6$$
  $\frac{1}{2}$  of  $6 = \bar{e}$ 

$$\frac{1}{2}$$
 of  $6 = \boxed{e}$ 

**9.** 
$$6 \div 3 = \overline{r}$$
  $\frac{1}{3}$  of  $6 = \overline{s}$ 

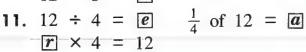
$$\frac{1}{3}$$
 of 6 = **s**

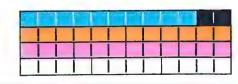
**10.** 
$$3 \times \overline{u} = 12 \frac{1}{3} \text{ of } 12 = \overline{v}$$

$$\frac{1}{3}$$
 of 12 =  $\bar{v}$ 

$$12 \div 3 = \mathbf{x}$$

$$\frac{1}{2}$$
 of 12 =  $\sqrt{a}$ 





There are two parts to every measurement, a unit of measure and a number called the measure.

Measure the line segments to the nearest quarter of an inch.

12. m 
$$(\overline{AB}) = \overline{n}$$

13. m 
$$(\overline{CD}) = \overline{z}$$

If 8 pints make 1 gallon, then

- 14. 1 pint makes a of a gallon. 15. 2 pints make c of a gallon.
- 16. 3 pints make e of a gallon. 17. 4 pints make v of a gallon.

188 (one hundred eighty-eight)

#### EXTRA FOR EXPERTS



1. Complete these equations.

**a.** Since 
$$34 + 34 + 34 + 34 = 136$$
,  $\frac{1}{4}$  of  $136 = \overline{a}$ 

**b.** Since 
$$476 + 476 + 476 = 1428$$
,  $\frac{1}{3}$  of  $1428 = \boxed{c}$ 

**c.** Since 
$$5 \times 396 = 1980$$
,  $\frac{1}{5}$  of  $1980 = \overline{n}$ 

**d.** Since 
$$\frac{1}{4}$$
 of 396 = 99, 4 × 99 =  $\mathbf{r}$ 

**e.** Since 
$$\frac{1}{4}$$
 of 2356 = 589,  $4 \times 589 = 5$ 

To use the code, you write the letter instead of the numeral. Check the addition and subtraction.

Make up other examples like this. Then have your neighbor check your work.

3. Find a short method for naming these sums.

$$7+3+3+3+3 = a$$
  $8+3+3+3+3 = s$   
 $6+4+4+4 = c$   $7+4+4+4 = m$   
 $6+5+5+5+5 = e$   $9+5+5+5+5 = u$   
 $3+2+2+2+2+2=r$   $18+2+2+2+2=x$ 

Just For Fun

4. Kate wrote 11 on a piece of paper. She used a pair of scissors to cut 11 and get two ones.

Sue wrote 11 on a piece of paper. She used a pair of scissors to cut 11 and get two elevens.

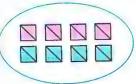
How did the two girls cut their papers?

5. If you have a dog whose weight is 6 pounds plus half its full weight, what is the weight of the dog?

# CHAPTER 7 • Multiplication and Division with Factors to 9



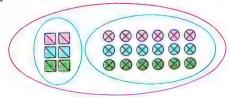
Study the sets and the equations.



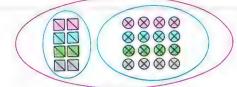


#### EXERCISES

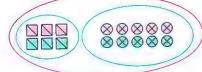
Study the sets and solve the equations.



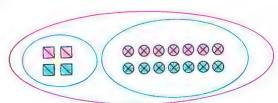
1.  $3 \times 2 = \overline{a}$ ;  $3 \times 6 = \overline{c}$ so  $3 \times 8 = \overline{e}$ 



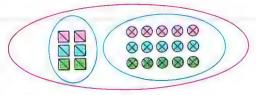
3.  $4 \times 2 = \overline{a}$ ;  $4 \times 4 = \overline{c}$ so  $4 \times 6 = \overline{e}$ 



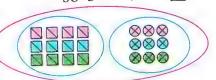
5.  $2 \times 3 = \mathbf{a}$ ;  $2 \times 5 = \mathbf{c}$ so  $2 \times 8 = \mathbf{e}$ 



2.  $2 \times 2 = \overline{a}$ ;  $2 \times 7 = \overline{e}$ so  $2 \times 9 = \overline{r}$ 



4.  $3 \times 2 = \overline{a}$ ;  $3 \times 5 = \overline{c}$ so  $3 \times 7 = \overline{e}$ 

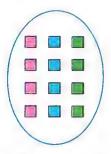


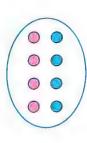
**6.**  $3 \times 4 = \mathbf{a}$ ;  $3 \times 3 = \mathbf{c}$  so  $3 \times 7 = \mathbf{e}$ 

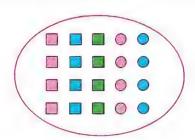
#### Joining equivalent sets



tudy the sets and the equations.

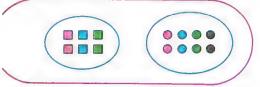


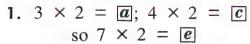


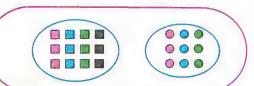


#### EXERCISES

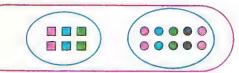
tudy the sets and solve the equations.



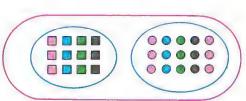




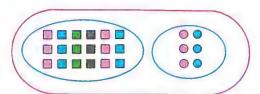
3. 
$$4 \times 3 = 5$$
;  $3 \times 3 = \overline{u}$   
so  $7 \times 3 = \overline{v}$ 



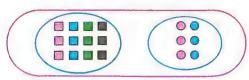
5. 
$$3 \times 2 = \overline{n}$$
;  $5 \times 2 = \overline{a}$   
so  $8 \times 2 = \overline{x}$ 



**2.** 
$$4 \times 3 = \overline{n}$$
;  $5 \times 3 = \overline{s}$  so  $9 \times 3 = \overline{r}$ 



**4.** 
$$6 \times 3 = \overline{v}$$
;  $2 \times 3 = \overline{x}$  so  $8 \times 3 = \overline{z}$ 



**6.** 
$$4 \times 3 = \mathbf{a}$$
;  $2 \times 3 = \mathbf{c}$  so  $6 \times 3 = \mathbf{e}$ 

(one hundred ninety-one) 191

#### Common factors

Bobby had 5 brown cars and 3 red cars. To find the number of wheels he needed for his cars, he wrote  $(5 \times 4) + (3 \times 4)$ . He renamed  $5 \times 4$  as 20 and  $3 \times 4$  as 12. He wrote these equations



$$(5 \times 4) + (3 \times 4) = 20 + 12$$
  
= 32

His sister said that since  $5 \times 4$  and  $3 \times 4$  both have the factor 4, he could have added 5 and 3 and then multiplied the sum by 4. She wrote these equations.

$$(5 \times 4) + (3 \times 4) = (5 + 3) \times 4$$
  
= 8 × 4  
= 32

A factor which occurs twice like the 4 in  $(5 \times 4)$  and  $(3 \times 4)$  is called a **common factor**.

#### EXERCISES

Copy and complete these pairs of equations.

**2.** 
$$(2 \times 6) + (3 \times 6) = \overline{v} + \overline{e}$$
  $(2 \times 6) + (3 \times 6) = (2 + 3) \times 6$   $= \overline{s} \times 6$   $= \overline{z}$ 

3. 
$$(2 \times 4) + (6 \times 4) = \overline{n} + \overline{u}$$
  $(2 \times 4) + (6 \times 4) = (\overline{r} + \overline{a}) \times 4$   $= \overline{n} \times 4$   $= \overline{s}$ 

**4.** 
$$(3 \times 6) + (4 \times 6) = \mathbf{a} + \mathbf{c}$$
  $(3 \times 6) + (4 \times 6) = (\mathbf{s} + \mathbf{r}) \times 6$   $= \mathbf{u} \times 6$   $= \mathbf{v}$ 

# The common factor rule



To name the products  $4 \times 3$  and  $5 \times 3$ , Karen set up her work like this:

She used these products to name the product  $9 \times 3$  like this:

Karen checked her work with the equations:



Karen said, "The common factor rule lets me add the 4 and 5 and then multiply by 3.

#### EXERCISES

Solve these equations.

1. 
$$3 \times (6 + 2) = (3 \times 6) + (3 \times 2)$$
  
=  $\boxed{a} + \boxed{c}$   
=  $\boxed{e}$ 

or

$$6 + 2 = \overline{n}$$

$$\times 3$$

$$18 + \overline{r} = \overline{s}$$

2. 
$$2 \times (3 + 5) = (2 \times 3) + (2 \times 5)$$
  
=  $u + v$   
=  $x$ 

or 
$$3 + 5 =$$
 $\hat{\iota} \times 2 \rightarrow$ 

3. 
$$9 \times (3 + 2) = (9 \times 3) + (9 \times 2)$$
  
=  $\boxed{a} + \boxed{e}$   
=  $\boxed{c}$ 

$$a + a = r$$

$$3 + 2 = 3$$

$$\times 9 \times 9$$

$$\boxed{m + x} = \boxed{2}$$

Write the numerals to complete these number patterns. The first one has been done for you.

4.	×	4	5	4	+	5
	6	24	30		54	

5. 
$$\times$$
 6 2 8 4  $a$   $c$   $e$ 

6. 
$$\times \ 3 \ 2 \ 5 \ 5 \ s \ n \ v$$

7. 
$$\times$$
 7 2 9 2  $u v r$ 

#### Six as a factor

Study these equations and number patterns.

$$\frac{(3 \times 6) = 18}{+ (2 \times 6)} = \frac{12}{5 \times 6} = \frac{30}{30}$$

10.

#### EXERCISES

Write the numerals to complete these equations.

1. 
$$5 + 1 = 6$$

$$\times 6 \times 6$$

$$\boxed{a + c = e}$$

2. 
$$5 + 2 = 7$$
  
  $\times 6 \times 6$   
  $a + c = e$ 

3. 
$$5 + 3 = 8$$
$$\times 6 \times 6$$
$$\boxed{a + c = e}$$

Write the numerals to complete these number patterns.

7. 
$$\begin{array}{c|c} \times & 6 \\ \hline 4 & a \\ \hline 2 & c \\ \hline 6 & r \end{array}$$

8.

$$\begin{array}{c|cc} \times & 6 \\ \hline 6 & x \\ \hline 3 & v \\ 9 & a \end{array}$$

Name the products.

12. 
$$(5 \times 6) = \boxed{a}$$
  
  $+ (1 \times 6) = \boxed{c}$   
  $6 \times 6 = \boxed{e}$ 

$$\frac{+(2 \times 6) = \boxed{c}}{7 \times 6 = \boxed{e}}$$
**16.**  $(6 \times 6) = \boxed{n}$ 

13.  $(5 \times 6) = a$ 

14. 
$$(5 \times 6) = \boxed{a}$$
  
  $+ (3 \times 6) = \boxed{c}$   
  $8 \times 6 = \boxed{e}$ 

15. 
$$(6 \times 6) = \boxed{a}$$

$$+ (1 \times 6) = \boxed{c}$$

$$7 \times 6 = \boxed{e}$$

16. 
$$(6 \times 6) = \overline{n}$$

$$+ (2 \times 6) = \overline{a}$$

$$8 \times 6 = \overline{v}$$

17. 
$$(6 \times 6) = \boxed{v}$$

$$+ (3 \times 6) = \boxed{e}$$

$$9 \times 6 = \boxed{r}$$

Use the fact that  $6 = 2 \times 3$  to solve these equations.

**18.** 
$$3 \times 3 = 9$$
, so  $6 \times 3 = \overline{a}$ 

**19.** 
$$3 \times 4 = 12$$
, so  $6 \times 4 = x$ 

**20.** 
$$3 \times 5 = 15$$
, so  $6 \times 5 = \boxed{c}$ 

**21.** 
$$3 \times 6 = 18$$
,  $so 6 \times 6 = \boxed{r}$ 

**22.** 
$$3 \times 7 = 21$$
, so  $6 \times 7 = \boxed{e}$ 

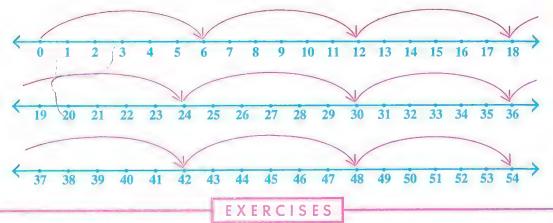
**23.** 
$$3 \times 8 = 24$$
, so  $6 \times 8 = 5$ 

**24.** 
$$3 \times 9 = 27$$
, so  $6 \times 9 = n$ 

**25.** 
$$3 \times 10 = 30$$
, so  $6 \times 10 = \overline{u}$ 

### Counting by sixes





Jse the number line to help you.

- 1. Complete this set: {0, 6, 12, 18, **a**, **c**, **e**, **n**, **r**, 54}
- 2. Name the number on the number line just after

$$a. 2 \times 6$$

**b.** 
$$5 \times 6$$

c. 
$$7 \times 6$$

d. 
$$8 \times 6$$

**3.** Name the number on the number line just before

$$a. 3 \times 6$$

**b.** 
$$4 \times 6$$

d. 
$$7 \times 6$$

4. Name the numbers on the number line between

**a.** 
$$4 \times 6$$
 and  $5 \times 6$ 

**b.** 
$$8 \times 6$$
 and  $9 \times 6$ 

Name the products.

6

Write the numerals to complete the chart.

12.	factor	6	n	6	0	6	6	r	S	6
	factor	a	6	c	6	e	z	6	6	u
	product	30	6	42	18	36	24	54	48	12



- **13.** How many in 1 package?
- 15. How many in 7 packages?



- 14. How many in 1 pack?
- 16. How many in 8 packs?

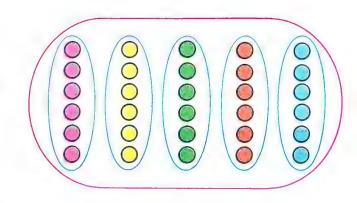
# Multiplication and division

5 sets of 6 form 1 set of 30.  $5 \times 6 = 30$ 

**Factor times Factor equals** Product.

1 set of 30 forms 5 sets of 6.  $30 \div 6 = 5$ 

**Product divided by Factor** equals Factor.



Another way to show **division** is to use the sign  $\Gamma$ 

capla hill

Factor Factor) Product

#### EXERCISES

Name the products.

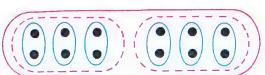
$$\frac{6}{2. \ 4)|n|}$$

4. 2) 
$$r$$

Name the missing factors.

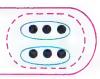
**8.** 
$$5)\overline{40}$$
 **9.**  $6)\overline{24}$  **10.**  $3)\overline{27}$  **11.**  $4)\overline{32}$  **12.**  $4)\overline{36}$ 

13. 
$$2)\overline{12}$$
 14.  $4)\overline{20}$  15.  $3)\overline{18}$  16.  $5)\overline{35}$  17.  $6)\overline{18}$  18.  $5)\overline{45}$  Study the sets, then name the missing factors.









19. 
$$a \times 2 = 12$$

**20.** 
$$12 \div 2 = c$$

**21**. 
$$e \times 6 = 12$$

**22.** 
$$12 \div 6 = n$$

**23.** 
$$r \times 6 = 18$$

**24.** 
$$18 \div 6 = \mathbf{z}$$

**25.** 
$$[s] \times 3 = 18$$

**26.** 
$$18 \div 3 = x$$

**30**. 6)18

#### Families of facts



	0		*		5
			*		5 5
			*		5
	•		*	_	5
	0	Δ	☆	$\nabla$	5
			*		5
6 +	- 6 -	- 6 -	- 6 -	- 6 =	= 30

$$+ 6 + 6 + 6 = 30$$

6 sets of 5 form 1 set of 30.

$$6 \times 5 = 30$$

5 sets of 6 form 1 set of 30.

$$5 \times 6 = 30$$

1 set of 30 forms 6 sets of 5.

$$30 \div 5 = 6$$

1 set of 30 forms 5 sets of 6.

$$30 \div 6 = 5$$

 $6 \times 5 = 30, 5 \times 6 = 30, 30 \div 6 = 5, 30 \div 5 = 6$ are a family of facts for the factors 6 and 5 and the product 30.

#### EXERCISES

Write a family of facts for each set of two factors and a product.

Name the missing factor or product.

9. 
$$3 \times |\bar{a}| = 27$$

10. 
$$4 \times 7 = c$$

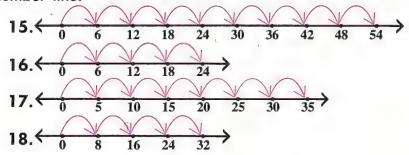
11. 
$$5 \times 5 = [e]$$

12. 
$$8 \times v = 24$$

13. 
$$|r| \times 4 = 36$$

14. 
$$9 \times 6 = [s]$$

Write four equations for the multiplication shown on each number line.



Name the products or missing factors.

19. 
$$2) \frac{a}{4}$$

**21.** 
$$3)\overline{12}$$
 **22.**  $3)\overline{n}$  **23.**  $3)\overline{21}$  **24.**  $3)\overline{24}$ 

$$\frac{0}{n}$$

**25.** 
$$4\overline{)16}$$
 **26.**  $4\overline{)12}$  **27.**  $4\overline{)28}$  **28.**  $5\overline{)15}$  **29.**  $5\overline{)25}$  **30.**  $6\overline{)36}$ 

(one hundred ninety-seven) 197

# Seven as a factor

Study these equations and number patterns.

$$3 + 2 = 5$$
  
 $\times 7 \times 7$   
 $21 + 14 = 35$ 

×	3	2	5
7	21	14	35

$$(3 \times 7) = 21$$

$$+ (2 \times 7) = 14$$

$$5 \times 7 = 35$$

#### EXERCISES

Write the numerals to complete these equations.

1. 
$$3 + 3 = 6$$

$$\times 7 \times 7$$

$$\boxed{a + c} = \boxed{e}$$

2. 
$$2 + 5 = 7$$
  
 $\times 7 \times 7$   
 $c + e = n$ 

3. 
$$2 + 6 = 8$$
  
 $\times 7 \times 7$   
 $n + r = \overline{s}$ 

4. 
$$(6 \times 7) = \boxed{a}$$
  
 $+ (1 \times 7) = \boxed{c}$   
 $7 \times 7 = \boxed{e}$ 

5.  $(6 \times 7) = \boxed{c}$   
 $+ (2 \times 7) = \boxed{e}$   
 $8 \times 7 = \boxed{a}$ 

5. 
$$(6 \times 7) = \boxed{c}$$

$$+ (2 \times 7) = \boxed{e}$$

$$8 \times 7 = \boxed{a}$$

6. 
$$(6 \times 7) = \boxed{x}$$
$$+ (3 \times 7) = \boxed{n}$$
$$9 \times 7 = \boxed{r}$$

Copy and complete this number pattern.

7.	1	8	15	29		43		57
	2	9						
	3	10						
	4	11						
	5	12						
	6	13					55	
	7	14			42			63

Use the number pattern to help you.

- 8. Name the number which is 1 greater than
  - $a. 8 \times 7$
- b.  $5 \times 7$
- c.  $6 \times 7$
- d.  $3 \times 7$
- **9.** Name the number which is 2 greater than

  - **a.**  $3 \times 7$  **b.**  $7 \times 7$
- c.  $6 \times 7$
- d.  $4 \times 7$
- 10. Name the number which is 3 greater than
  - $a. 6 \times 7$
- **b.**  $8 \times 7$  **c.**  $7 \times 7$
- d.  $5 \times 7$

- 11. Name the number which is 1 less than
  - $\alpha$ .  $3 \times 7$
- **b.**  $5 \times 7$  **c.**  $6 \times 7$
- d.  $4 \times 7$

# Multiplication-division with seven





28 days in 4 weeks How many days in 1 week?  $28 \div 4 = 7$ 



7 plants in 1 box How many plants in 4 boxes?  $4 \times 7 = 28$ 

#### EXERCISES

Write the numerals to complete this table.

On the top row count back by sevens from 63. On the bottom row name the number of sevens you have subtracted.

J.	63	56	49	42	a	$\boldsymbol{c}$	e	n	r	S
		1	2	.3	S	u	v	$\boldsymbol{x}$	z	C

Write the numerals to complete these sentences.

- **2.** 6 sets of 7 form 1 set of **C**.
- **4.** 7 sets of 7 form 1 set of **@**.
- **6.** 8 sets of 7 form 1 set of **n**.
- **8.** 9 sets of 7 form 1 set of **a**.
- **3.** A set of 42 forms  $\mathbf{r}$  sets of 7.
- **5.** A set of 49 forms sets of 7.
- 7. A set of 56 forms  $\overline{v}$  sets of 7.
- **9.** A set of 63 forms **Z** sets of 7.

Name the products or missing factors.

17. 
$$7 \times \boxed{a} = 42$$
  $6 \times 7 = \boxed{n}$   $42 \div 6 = \boxed{c}$   $42 \div 7 = \boxed{s}$ 

**18.** 
$$\mathbf{e} \times 7 = 56$$
  $7 \times \mathbf{o} = 56$   $56 \div 7 = \mathbf{a}$   $56 \div \mathbf{r} = 7$ 

**19.** 
$$7 \times \overline{u} = 63$$
  $\overline{s} \times 7 = 63$   $63 \div 7 = \overline{r}$   $63 \div \overline{v} = 7$ 

**20.** 
$$7 \times \mathbf{z} = 49$$
 **n**  $\times 7 = 49$   $49 \div 7 = \mathbf{o}$   $49 \div \mathbf{s} = 7$ 

21. 
$$7)35$$
 22.  $7)63$  23.  $7)56$  24.  $7)n$  25.  $7)e$  26.  $7)u$ 

# Eight as a factor

Study these equations.

$$5 \times 8 = 40$$
, and  $1 \times 8 = 8$ , so  $6 \times 8 = 40 + 8 = 48$   
and  $2 \times 8 = 16$ , so  $7 \times 8 = 40 + 16 = 56$   
and  $3 \times 8 = 24$ , so  $8 \times 8 = 40 + 24 = 64$   
and  $4 \times 8 = 32$ , so  $9 \times 8 = 40 + 32 = 72$ 

#### EXERCISES

Name the products.

1. 
$$0 \times 8 = a$$

**2.** 
$$2 \times 8 = c$$

3. 
$$3 \times 8 = e$$

4. 
$$5 \times 8 = \overline{n}$$
 5.  $6 \times 8 = \overline{0}$  6.  $8 \times 8 = \overline{r}$ 

5. 
$$6 \times 8 = 6$$

6. 
$$8 \times 8 = r$$

Write the numerals to complete these number patterns.

13. Count by eights to 80. Make a number pattern for 8 like the one in exercise 7 on page 198.

Use the number pattern for 8 to help you.

14. Name the number which is 1 greater than

$$a. 2 \times 8$$

c. 
$$5 \times 8$$

d. 
$$6 \times 8$$

15. Name the number which is 2 greater than

**c.** 
$$9 \times 8$$

16. Name the number which is 3 greater than

$$\mathbf{a}. 3 \times 8$$

c. 
$$7 \times 8$$

17. Name the number which is 1 less than

$$\mathbf{a}. 4 \times 8$$

**b.** 
$$5 \times 8$$
 **c.**  $3 \times 8$ 

d. 
$$7 \times 8$$

Name the products or missing factors.

18. 5 19. 6 20. 8 21. 8 22. 8 
$$\times 8$$
  $\overline{a}$   $\times 8$   $\overline{c}$   $\overline{e}$   $\overline{e}$   $\overline{n}$   $\overline{n}$   $\overline{72}$ 

$$\frac{\times 9}{|e|}$$

|n|

22. 
$$8 \times \boxed{r}$$

$$\begin{array}{c} \mathbf{23.} \quad \boxed{\$} \\ \times 8 \\ \hline \end{array}$$

64

#### Nine as a factor



Study these equations.

$$5 \times 9 = 45$$
, and  $1 \times 9 = 9$ , so  $6 \times 9 = 45 + 9 = 54$   
and  $2 \times 9 = 18$ , so  $7 \times 9 = 45 + 18 = 63$   
and  $3 \times 9 = 27$ , so  $8 \times 9 = 45 + 27 = 72$   
and  $4 \times 9 = 36$ , so  $9 \times 9 = 45 + 36 = 81$ 

#### EXERCISES

Name the products.

1. 
$$4 \times 9 = e$$

2. 
$$5 \times 9 = n$$

3. 
$$6 \times 9 = [a]$$

4. 
$$7 \times 9 = [r]$$

5. 
$$8 \times 9 = 5$$

**6.** 
$$9 \times 9 = u$$

Write the numerals to complete these number patterns.

7. 
$$\times$$
 9 4  $n$  1  $x$  5  $r$ 

9. 
$$\begin{array}{c|c} \times & 9 \\ \hline 6 & x \\ \hline 1 & z \\ \hline 7 & a \end{array}$$

$$\begin{array}{c|c}
11. & \times & 9 \\
\hline
8 & n \\
\hline
1 & x \\
9 & r
\end{array}$$

13. Count by nines to 90. Make a number pattern for 9 as you did for 7 and 8.

Use your number pattern for 9 to help you.

14. Name the number which is 1 greater than

$$\mathbf{q}$$
,  $4 \times 9$ 

d. 
$$5 \times 9$$

15. Name the number which is 2 greater than

16. Name the number which is 3 greater than

$$\mathbf{a.} \ 3 \times 9$$

**b.** 
$$4 \times 9$$
 **c.**  $8 \times 9$ 

c. 
$$8 \times 9$$

**d.** 
$$9 \times 9$$

u

 $\times 9$ 

23.

Name the products or missing factors.

- 18. 6 19. 9 20. m 21. 9 22. 17.  $\times$  0 × 9  $\times$  9 × 9  $\times x$ 
  - 27 45 63 36 |a||c|e 6 **27.** 4)36 **28.** 5)45 **29.** 7)63 **24.** 9) a **25.** 9) c **26.** 9) e

9

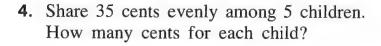
 $\times [r]$ 

#### Using sets and numbers

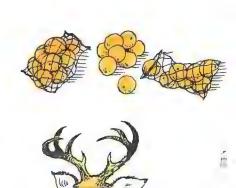
1. How many teams of 5 horses can be made from 45 horses?



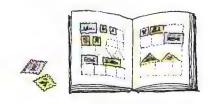
- **2.** One box contains 4 cakes of soap. How many cakes are there in 8 boxes?
- **3.** Each bag can hold 9 oranges, and there are 54 oranges in all. How many bags can be filled?



- **5.** If each deer has 8 points on its antlers, then how many points would 7 deer have on their antlers?
- **6.** If you have 36 marbles to share evenly among 4 boys, how many marbles will each boy receive?
- 7. There are 6 players on a hockey team. How many players in all are there on 8 hockey teams?
- **8.** If you can put 8 stamps on each page, then how many stamps can you put on 7 pages?
- **9.** If each toy costs 8¢, how much will 9 toys cost?





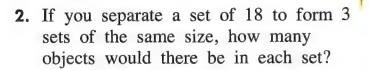




### Using sets and numbers



1. There are 74 children in this room and 26 in another room. How many children are there in the two rooms?





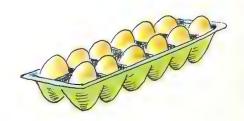
- **3.** There are 12 inches in a foot. How many inches are there in 6 feet?
- **4.** Bob weighs 83 pounds. Bill weighs 67 pounds. How many pounds heavier is Bob than Bill?



- **5.** There are 24 hours in 1 day. How many hours are there in 2 days?
- **6.** There are 16 ducks on the pond. Each duck has 2 legs. How many legs are there in all?

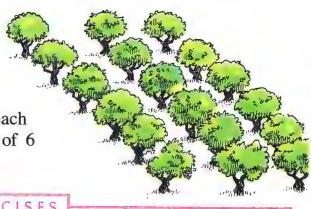


- **7.** A week is 7 days. How many days are there in 8 weeks?
- **8.** There are 12 things in a dozen. How many things are there in 3 dozen?
- **9.** Every car holds 6 children. How many children in all are there in 8 cars?



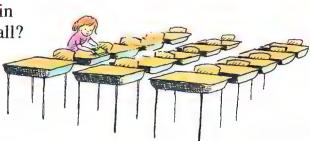
### Multiplication again

There are 3 rows with 6 trees in each row. There are 3 sets of 6, 3 sets of 6 make 1 set of 18.  $3 \times 6 = 18$ .



#### EXERCISES

1. There are 3 rows with 5 desks in each row. How many desks in all?



(1,6) 6th St.

2. There are 3 rivers and 4 roads. How many bridges are there?

**3.** There are 5 avenues and 6 streets. In how many points do the avenues and streets meet?

4. In question 3, the points where First Avenue meets the Streets may be named (1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6). Name the points

a. where Second Avenue meets the Streets

**b.** where Fourth Avenue meets the Streets.

(1,5)
(1,4)
4th St.

(1,3)
3rd St.

(1,2)
2nd St.

(1,1)
(1,1)
1st St.

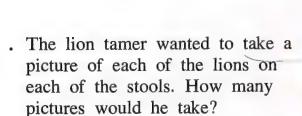
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#### **Pairing**



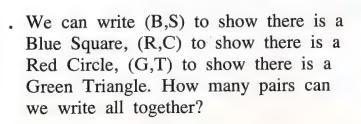
- . Each of the children can sit at any of the desks. To show that Harry can sit at any desk we shall write (H,1) (H,2) (H,3).
  - a. Write the pairs to show that Mary and Polly can sit at any desk.
  - **b.** How many pairs can you write in all?



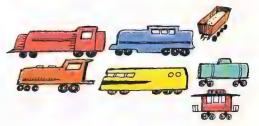


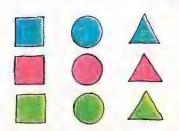


- . Each of the boys can wear any of the costumes. How many different pictures can you take showing a boy wearing a costume?
- . Each of the cars will fit behind any of the engines. How many different pairs of engines and cars can you make?



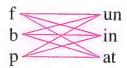






# **Pairing**

Carmen and Ann were playing a Word-Making Game. To play the game they would start with two sets of letters and see how many words they could make. For example, they had the set of letters {f, b, p}. These had to be the first letters in the words they made. They also had the pairs of letters {un, in, at}. They paired each of the letters in the first set with each of the pairs of letters in the second set, like this:



How many pairings did the girls make? Write the words they made.

#### EXERCISES

How many pairings can you make with the members of each of these pairs of sets? First write the number of pairings. Then write the words you can make. Remember, the letters in the first set must come first in your words.

1.  $\{i, o\}$   $\{f, n\}$ 

2.  $\{a, i\}$   $\{t, n\}$ 

**3.**  $\{l, h, b, m\}$  {ad, op}

- **4.** {1, b, s} {and, end}
- 5.  $\{b, r, f\}$  {an, ed, at, oot}
- **6.**  $\{d, h, l\}$   $\{og, id\}$

7.  $\{t, 1, h\}$  {ip, ot, ow}

8.  $\{f, h, b\}$   $\{it, at\}$ 





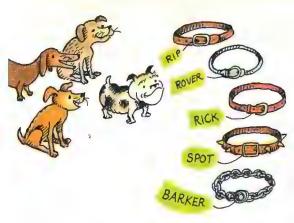








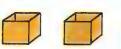
9. Karen had cut out 3 pictures of doll heads and 4 pictures of doll dresses. How many dolls can she make by pairing the heads and the dresses?



1. How many ways can the puppies be paired with the names on the right?

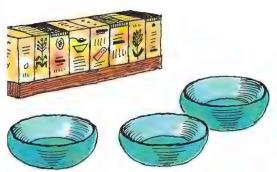








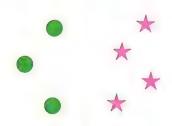
2. Each of the balls can fit in each of the boxes. How many possible pairings are there?



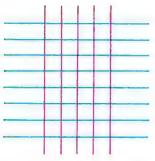
**3.** How many ways can the cereal be put into the bowls?



4. How many ways can the 4 paper dolls be dressed?



5. Each of the circles can be paired with each of the stars. How many possible pairings are there?

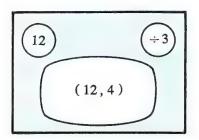


6. 8 blue line segments, 5 red line segments. How many points are there where a blue segment and a red segment meet?

#### Number pairs and number patterns

Our Number Pair TV can use division rules.

Turn the number knob to 12 and tune in the rule **divide by 3**,  $12 \div 3 = 4$ . The screen shows the number pair (12, 4).



#### EXERCISES

Think of a multiplication or division rule to complete these sets of number pairs.

**1.** 
$$\{(2, 10), (3, 15), (4, \overline{a}), (6, \overline{x}), (7, \overline{c}), (9, \overline{z})\}$$

**2.** 
$$\{(3,24), (6,48), (5, e), (4, a), (2, r), (1, n)\}$$

**3.** 
$$\{(2,18), (3,27), (4, \mathbf{S}), (7, \mathbf{a}), (6, \mathbf{v}), (5, \mathbf{e})\}$$

**4.** 
$$\{(18,2), (27,3), (9, \mathbf{x}), (36, \mathbf{n}), (81, \mathbf{r}), (54, \mathbf{u})\}$$

**5.** 
$$\{(12,2), (24,4), (18, a), (30, r), (36, s), (54, n)\}$$

Complete these equations.

6. 
$$2 + 4 = 6$$
  
 $\times 9$   $\times 9$   
 $a + r = c$ 

7. 
$$4 + 5 = 9$$
  
 $\times 9 \times 9$   
 $\boxed{r + e} = \boxed{x}$ 

8. 
$$3 + 6 = 9$$
  
 $\times 7 \times 7$   
 $s + u = e$ 

Write the numerals to complete these patterns.

10. 
$$\times$$
 5 4 5 + 4 4 20  $v$   $c$ 

11. 
$$\times 363+6$$
5 15  $r$   $e$ 

12. 
$$\times$$
 5 4 5 + 4 6 30 s n

13. 
$$\times$$
 4 6 4 + 6 7 28  $a$   $x$ 

7	20.	×	6
s		6	v
e		4	S
u		10	x

### Multiplication with tens



ame these products.

1. 
$$3 \times 10 = a$$

**2.** 
$$4 \times 10 = C$$

3. 
$$6 \times 10 = e$$

4. 
$$3 \times 20 = n$$

5. 
$$4 \times 20 = \bar{a}$$

6. 
$$6 \times 20 = r$$

7. 
$$3 \times 30 = s$$

8. 
$$4 \times 30 = u$$

**9.** 
$$6 \times 30 = v$$

10. 
$$3 \times 2 \times 10 = \overline{a}$$
  
  $3 \times 20 = \overline{c}$ 

11. 
$$3 \times 4 \times 10 = e$$

**12.** 
$$4 \times 6 \times 10 = C$$

$$3 \times 40 = \boxed{n}$$

$$4 \times 60 = \boxed{r}$$

'rite the numerals to complete these number patterns.

### EXTRA FOR EXPERTS



### A multiplication pattern

ĺΧ	2	4	6	8	1	3	5	7	9
2×	6	2	8	4	5				9
3×	4							9	
1×	8	6	4	2	5	3	1	9	7
5×	2	4	6	8	5	7	9	1	3
7×					3				7
3×	4	8	2	6	5	9	3	7	1
9×	8	6	4	2	9	7	5	3	1

On our chart we have written factors. Draw a chart like ours, writing products in place of the factors. The third row would look like this:

3× 12 24 6 18 21 3 15 27 9

Can you see the pattern?

olve these equations.

17. 
$$(4 \times 5) = \mathbf{a}$$
  
 $(4 \times 5) = \mathbf{c}$ 

$$\begin{array}{c} + (2 \times 5) = \boxed{e} \\ \hline 10 \times 5 = \boxed{v} \end{array}$$

18. 
$$(6 \times 3) = \overline{u}$$
$$(3 \times 3) = \overline{e}$$
$$+ (1 \times 3) = \overline{r}$$
$$10 \times 3 = \overline{a}$$

19. 
$$(5 \times 7) = \overline{v}$$
  
 $(3 \times 7) = \overline{e}$   
 $+ (2 \times 7) = \overline{r}$   
 $10 \times 7 = \overline{a}$ 

### Two-place multiplication

Study these methods for naming the product  $3 \times 12$ .

### **Equations**

$$3 \times 12 = 3 \times (10 + 2)$$
  
=  $(3 \times 10) + (3 \times 2)$   
=  $30 + 6$   
=  $36$ 

### **Expanded numerals**

$$\begin{array}{c}
 12 = 10 + 2 \\
 \times 3 & \times 3 \\
 \hline
 30 + 6 = 36
 \end{array}$$

# Number patterns

×	10	2	12
3	30	6	36

#### EXERCISES

Write the numerals to complete these equations.

1. 
$$2 \times 14 = 2 \times (10 + 4)$$
  
=  $(2 \times 10) + (2 \times 4)$   
=  $\boxed{a} + \boxed{c}$   
=  $\boxed{e}$ 

3. 
$$5 \times 11 = 5 \times (10 + 1)$$
  
=  $(5 \times 10) + (5 \times 1)$   
=  $\boxed{a} + \boxed{c}$   
=  $\boxed{e}$ 

5. 
$$4 \times 12 = 4 \times (10 + 2)$$
  
=  $(4 \times \boxed{a}) + (4 \times \boxed{c})$   
=  $\boxed{e} + \boxed{n}$   
=  $\boxed{o}$ 

2. 
$$3 \times 13 = 3 \times (10 + 3)$$
  
=  $(3 \times 10) + (3 \times 3)$   
=  $\boxed{a} + \boxed{c}$   
=  $\boxed{e}$ 

4. 
$$3 \times 11 = 3 \times (10 + 1)$$
  
=  $(3 \times 10) + (3 \times 1)$   
=  $\boxed{a} + \boxed{c}$   
=  $\boxed{e}$ 

6. 
$$8 \times 11 = 8 \times (10 + 1)$$
  
=  $(8 \times 10) + (8 \times 1)$   
=  $\boxed{a} + \boxed{c}$   
=  $\boxed{e}$ 

Write the numerals to complete these examples.

7. 
$$13 = 10 + 3$$
  
 $\times 2$   
 $\boxed{a + c} = \boxed{e}$ 

8. 
$$12 = 10 + 2$$
  
 $\times 2$   
 $\boxed{a + c} = \boxed{e}$ 

9. 
$$11 = 10 + 1$$
  
 $\times 4$   
 $\boxed{a + c} = \boxed{e}$ 

Write the numerals to complete these number patterns.

10. 
$$\times 10 \ 2 \ 12$$
  $2 \ a \ c \ e$ 

11. 
$$\times$$
 10 1 11 8 a c e

12. 
$$\times 10 | 3 | 13$$
  
2 s u v

13. 
$$\times 10 \ 1 \ 11$$
  $7 \ x \ z \ a$ 

15. 
$$\times 10 \ 3 \ 13$$
  $3 \ n \ o \ r$ 

16. 
$$\times 10 \ 1 \ 11$$
 $4 \ s \ u \ v$ 

17. 
$$\times 10 \ 1 \ 11$$
 9  $x \ z \ a$ 

## Two-place multiplication



Study these methods for naming the product  $3 \times 12$ .

Equations	Three- step method	The Short method
$3 \times 12 = 3 \times (10 + 2)$ = $(3 \times 10) + (3 \times 2)$ = $30 + 6$ = $36$		$ \begin{array}{r} 12 \\ \times 3 \\ \hline 36 \text{ Step 1: } 3 \times 2 = 6 \\ \text{ Step 2: } 3 \times 10 = 30 \\ \text{Write 6 in the ones' place} \\ \text{and 3 in the tens' place.} \end{array} $

### EXERCISES

se the 3-step method to name these products.

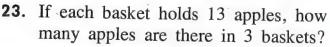
**8 -14.** Use the short method to name the products in examples 1-7.

ame the missing factors.

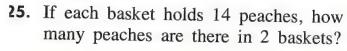
15. 12 16. 13 17. 11 18. 12 19. 11 20. 12 21. 13 
$$\times \boxed{a}$$
  $\times \boxed{c}$   $\times \boxed{c}$   $\times \boxed{e}$   $\times \boxed{e}$   $\times \boxed{n}$   $\times \boxed{v}$   $\times \boxed{v}$   $\times \boxed{r}$   $\times \boxed{s}$   $\times \boxed{s}$ 

lve the problems.

22. If there are 7 rows of apple trees with 11 trees in each row, how many apple trees are there altogether?



24. If there are 4 rows of peach trees with 12 trees in each row, how many peach trees are there altogether?









# Multiplying numbers between ten and twenty

Study these methods for naming the product  $3 \times 16$ .

### **Equations**

$$3 \times 16 = 3 \times (10 + 6)$$
  
=  $(3 \times 10) + (3 \times 6)$   
=  $30 + 18$   
=  $48$ 

### **Expanded numerals**

$$\begin{array}{c}
 16 = 10 + 6 \\
 \times 3 & \times 3 \\
 \hline
 30 + 18 = 48
 \end{array}$$

### Number patterns

### EXERCISES

Write the numerals to complete these equations.

1. 
$$2 \times 16 = 2 \times (10 + 6)$$
  
=  $(2 \times 10) + (2 \times 6)$   
=  $\boxed{a} + \boxed{c}$   
=  $\boxed{e}$ 

3. 
$$3 \times 14 = 3 \times (10 + 4)$$
  
=  $(3 \times 10) + (3 \times 4)$   
=  $\boxed{a}$  +  $\boxed{c}$   
=  $\boxed{e}$ 

2. 
$$3 \times 15 = 3 \times (10 + 5)$$
  
=  $(3 \times 10) + (3 \times 5)$   
=  $\boxed{a} + \boxed{c}$   
=  $\boxed{e}$ 

**4.** 
$$3 \times \boxed{0} = 3 \times (10 + 7)$$
  
=  $(3 \times 10) + (3 \times 7)$   
=  $\boxed{a} + \boxed{c}$   
=  $\boxed{e}$ 

Write the numerals to complete these examples.

5. 
$$14 = 10 + 4$$
  
 $\times 5$   $\times 5$   $\boxed{a + c} = e$ 

6. 
$$12 = 10 + 2$$
  
 $\times 6$   $\times 6$   $\boxed{a + c} = e$ 

7. 
$$13 = 10 + 3$$

$$\times 5 \times 5$$

$$\boxed{a + c} = e$$

8. 
$$12 = 10 + 2$$
  
 $\times 7$   $\times 7$   
 $a + c = e$ 

$$\begin{array}{ccc}
18 &= 10 + 8 \\
\times 3 & \times 3 \\
\hline
\boxed{a} &+ \boxed{c} &= \boxed{e}
\end{array}$$

8. 
$$12 = 10 + 2$$
  
 $\times 7$   
 $a + c = e$ 

9.  $18 = 10 + 8$   
 $\times 3$   
 $a + c = e$ 

10.  $15 = 10 + 5$   
 $\times 5$   
 $a + c = e$ 

Write the numerals to complete these number patterns.

12. 
$$\times$$
 10 5 15 4  $n$   $v$   $r$ 

13. 
$$\times$$
 10 4 14 5  $r$  s  $u$ 

14. 
$$\times$$
 10 6 16 6  $v \times z$ 

15. 
$$\times$$
 10 7 17 2 a c e

16. 
$$\times$$
 10 3 13 7  $v \times z$ 

### Multiplication — short method



Study these methods for naming the product  $3 \times 16$ .

### Three-step method

**Step 3:** 30 + 18 = 48

### Short method

Step 1:  $3 \times 6 = 18$ Step 2:  $3 \times 10 = 30$ 

18 + 30 = 48

### EXERCISES

Write the numerals to complete these examples of the hree step method.

48

$$\frac{\times 4}{12}$$
40

U

$$\frac{\times 5}{15}$$

**2**. 13

$$\frac{c}{e}$$

$$\frac{\times 3}{24}$$

$$\frac{u}{c}$$

$$\frac{\times 6}{z}$$

$$\frac{\overline{60}}{\overline{a}}$$

$$\frac{\times 7}{\boxed{v}}$$

Jse the 3-step method to name these products.

7. 
$$5 \times 12 = |a|$$

8. 
$$8 \times 12 = [n]$$

9. 
$$4 \times 18 = C$$

10. 
$$5 \times 16 = r$$

11. 
$$6 \times 15 = [e]$$

12. 
$$7 \times 12 = \bar{z}$$

13. 
$$8 \times 13 = v$$

14. 
$$7 \times 13 = |x|$$

15. 
$$4 \times 17 = \overline{v}$$

16 -24. Use the short method to name the products in examples 7-15.

solve the problems.

- 25. If each box of a building kit contains 17 pieces, how many pieces are in 5 boxes?
- **26.** If you need 18 building pieces to build a house, how many pieces will you need to build 3 houses?

### Measurement — a yard



There are 12 inches in 1 foot.



There are 3 feet in 1 yard

### EXERCISES

#### Complete the sentences.

- 1. If 1 foot matches 12 inches, then 2 feet match <u>a</u> inches.
- 2. If 1 yard matches 3 feet, then 4 yards match c feet.

### How many inches are there in:

**3**. 2 feet

**4.** 3 feet

**5**. 4 feet

**6.** 5 feet

#### How many feet are there in:

**7.** 2 yards

**8.** 3 yards

**9.** 4 yards

**10.** 5 yards

#### Answer the questions.

- 11. Bob said the width of the classroom was 12 yards. Barbie said the width was 36 feet. Was this the same length?
- 12. Bill said the width of his desk was 34 inches. Jane said the width was 1 yard. Was this the same length?
- **13.** Would it be better to use yards or feet to measure
  - a. the height of a girl
  - **b.** a football field
  - c. the length of a broom
- 14. A foot contains 12 inches, so 3 feet contain **n** inches.

#### 214 (two hundred fourteen)

### Twelve in measurement



Lola looked at the height scale and said, I am 4 feet 2 inches tall." To find how nany inches tall she was, Lola thought: There are 12 inches in 1 foot, so  $\times$  12 = 48 and 48 + 2 = 50. Lola ound she was 50 inches tall.



#### EXERCISES

se the short method to name the products.

- 1.  $2 \times 12$
- **2.** 3 × 12
- **3**. 4 × 12.
- **4.**  $5 \times 12$

- 5. 6 × 12
- **6.**  $7 \times 12$  **7.**  $8 \times 12$
- 8. 9 × 12

o work with feet and inches you need to be able to ultiply 12.

ow many inches are there in

9. 2 feet

10. 3 feet

**11.** 4 feet

**12.** 5 feet

13. 6 feet

14. 7 feet

- **15.** 2 feet 3 inches
- **16.** 3 feet 4 inches
- **17.** 2 feet 9 inches

- **18.** 3 feet 6 inches
- **19.** 4 feet 6 inches
- 20. 5 feet 6 inches

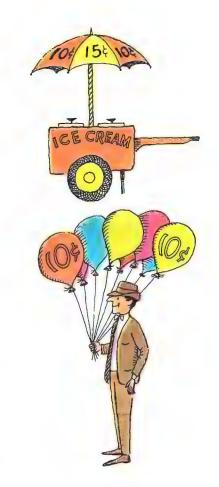
nswer the questions.

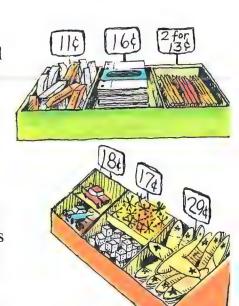
There are 60 minutes in one hour.

- 21. How many sets of 5 are there in a set of 60?
- **22.** How many numerals are there on a clock?
- 23. How many minutes are there between two numerals on a clock?
- 24. From 12 o'clock in the middle of the day until 12 o'clock in the middle of the night there are [a] hours.
- **25.** In a whole day there are  $\overline{v}$  hours.
- **26.** In two full days there are **C** hours.

### Using sets and numbers

- 1. Sam told Judy and Sally he would buy each of them a 15¢ ice cream cone. If he also buys one for himself, how much will it cost him in all?
- 2. Sally wanted to buy balloons for her little brother at home, and for Judy and Sam. If she also buys one for herself, how much will it cost her in all?
- **3.** Judy needs notebook paper. A pack of notebook paper costs 16¢. How much will she have to pay for 3 packs?
- 4. Sally needs erasers. Each eraser costs 11¢. She wants to buy two for herself and two for her brother. How much will it cost her?
- 5. If two pencils cost 13¢, how much will 8-pencils cost?
- 6. The children decide to buy themselves a set of jacks. If a set of jacks costs 17¢, how much will three sets cost?
- 7. Judy has four little brothers. She wants to buy them each a toy car which costs 18¢. How much will Judy have to pay?
- 8. If a model plane costs 29¢, how much will 4 planes cost?





### LOOKING WAY BACK



Write a 3- or 4- digit numeral for:

- 1. three hundred forty-six
- 2. four thousand six hundred twenty
- 3. five thousand nine hundred twenty-six

Write the numeral for:

- 4. 6 thousands, 5 hundreds, 2 tens, 7 ones
- **5.** 8 thousands, 9 hundreds, 0 tens, 0 ones
- 6. 7 thousands, 3 tens, 5 ones
- 7. 4 thousands, 8 hundreds, 9 ones
- 8. 5 thousands, 9 hundreds
- **9.** Using the digits 7, 6, 8, 2, what is the greatest counting number and the least counting number that can be shown?

Write these numerals in the short form.

10. 
$$300 + 50 + 7$$

11. 
$$6000 + 400 + 50 + 2$$

12. 
$$6000 + 200$$

Vrite the set which

- 14. contains the counting numbers greater than 6 and less than 10.
- 15. contains the first 5 letters of the alphabet.
- 16. is made by joining {A, B, C} and {D, E, F, G}.
- 17. is made by joining  $\{\Box, \bigcirc, \triangledown\}$  and  $\{\diamondsuit, \not \Rightarrow, \triangle\}$ .

copy and complete the examples.

18.
 463
 19.
 629
 20.
 639
 21.
 386
 22.
 264

 
$$+ 265$$
 $+ 286$ 
 $+ 249$ 
 $+ 283$ 
 $+ 578$ 
 $120$ 
 $100$ 
 $\boxed{m}$ 
 $160$ 
 $\boxed{v}$ 
 $600$ 
 $\boxed{800}$ 
 $\boxed{x}$ 
 $\boxed{s}$ 
 $\boxed{x}$ 
 $\boxed{a}$ 
 $\boxed{c}$ 
 $\boxed{r}$ 
 $\boxed{u}$ 
 $\boxed{z}$ 

### Interesting number facts

### Did you know?

- 1. The largest state in the Union (Alaska) is almost 500 times as large as the smallest (Rhode Island).
- 2. There are nearly 4 times as many people in Rhode Island as there are in Alaska.
- **3.** The population of the United States by 1966 was 3 times as great as it was in 1890.
- **4.** The diameter of the planet Jupiter is 11 times the diameter of the earth.
- **5.** A year on the planet Pluto is almost 250 times as long as our year.
- **6.** The sun is 400,000 times as bright as the full moon.
- 7. A block of black ironwood (the heaviest wood known) weighs 31 times as much as the same size block of balsa wood.
- **8.** You probably weigh about 9 times as much now as you did when you were born.
- **9.** You would weigh 3 times as much on Jupiter as you do on Earth.

### CHECKING UP



### Name these products.

**4.** 
$$8 \times 8$$

**5.** 
$$4 \times 9$$

### Solve these equations.

**9.** 
$$3 \times a = 27$$

10. 
$$8 \times [c] = 72$$

11. 
$$7 \times [0] = 49$$

12. 
$$e \div 9 = 4$$

13. 
$$r \div 6 = 7$$

14. 
$$u \div 3 = 8$$

15. 
$$24 \div 3 = v$$

**15.** 
$$24 \div 3 = \overline{v}$$
 **16.**  $35 \div 7 = \overline{z}$ 

17. 
$$48 \div 6 = [a]$$

18. 
$$35 \div s = 5$$

19. 
$$81 \div 9 = 0$$

**20.** 
$$9 \div e = 9$$

# Write the numerals to complete these number patterns.

23. 
$$\times$$
 6 2 8 9  $v$   $n$   $z$ 

24. 
$$\times$$
  $r$  4 7 9 27  $c$   $a$ 

25. 
$$\times$$
  $v$   $r$  7 7 35 14  $s$ 

### lame these products.

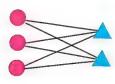
**27.** 
$$3 \times 11$$
 **28.**  $7 \times 12$  **29.**  $3 \times 14$ 

**28.** 
$$7 \times 12$$

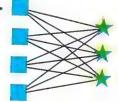
**31.** 
$$4 \times 15$$

ow many pairings are shown in each of these pictures?

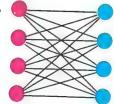
35.



36.



37.



- e your ruler if you need help to name the answers.
- 38. Name the number of inches in
  - **a.** 6 feet **b.** 8 feet **c.** 10 feet
- 39. Name the number of yards in
  - **a.** 6 feet **b.** 12 feet **c.** 15 feet
- 10. Name the number of feet in
  - **a.** 2 yards **b.** 3 yards **c.** 9 yards

In this chapter you learned 10 more multiplication facts.

Name the products.

3. 
$$7 \times 7$$

5. 
$$8 \times 7$$

10. 9 × 9

You learned that six of these equations belong in families of four facts. For example,  $7 \times 6 = 42$  goes with  $6 \times 7 = 42$ ,  $42 \div 6 = 7$ ,  $42 \div 7 = 6$ .

Write the other three equations for each of these.

11. 
$$9 \times 6 = 54$$

12. 
$$8 \times 7 = 56$$

13. 
$$9 \times 7 = 63$$

Four of the equations belong in families of two facts. For example,  $6 \times 6 = 36$  goes with  $36 \div 6 = 6$ .

Write the other equation for each of these.

14. 
$$7 \times 7 = 49$$

15. 
$$8 \times 8 = 64$$

**16.** 
$$9 \times 9 = 81$$

You used the common factor rule many times. This is the rule which says for example, that  $3 \times (5 + 4) = (3 \times 5) + (3 \times 4)$ . To put it in another way, if you want to find the product  $3 \times (5 + 4)$ , you may rename 5 + 4 as 9, then multiply  $3 \times 9 = 27$ ; or you may rename  $3 \times 5$  as 15 and  $3 \times 4$  as 12, then add 15 + 12 = 27.

Use the common factor rule to complete these equations.

17. 
$$5 \times 6 = 30$$
,  $4 \times 6 = 24$ , so  $\boxed{a} \times 6 = \boxed{c}$ 

**18.** 
$$5 \times 9 = 45$$
,  $4 \times 9 = 36$ , so  $\boxed{n} \times 9 = \boxed{0}$ 

19. 
$$8 \times 5 = 40$$
,  $8 \times 4 = 32$ , so  $8 \times \mathbf{r} = \mathbf{s}$ 

**20.** 
$$5 \times 10 = 50$$
,  $5 \times 4 = 20$ , so  $5 \times \bar{x} = \bar{z}$ 

### EXTRA FOR EXPERTS



You know that 
$$10 - 1 = 9$$
, so  $2 \times 9 = 2 \times (10 - 1)$   
=  $(2 \times 10) - (2 \times 1)$   
=  $20 - 2$   
=  $18$ 

Jse this method to find these products.

1. 
$$2 \times 9$$

**2.** 
$$3 \times 9$$

**3**. 
$$4 \times 9$$

**7.** 
$$8 \times 9$$

**9.** Write down the products with 9 as a factor. Add the digits. What do you notice? Look at the ones' digit in the products. What do you notice? Try this also for the products with 3 as a factor.

Complete these equations.

10. 
$$5 \times 6 = \overline{a} \times 5 = 2 \times \overline{c} = \overline{e}$$
 11.  $7 \times 6 = \overline{n} \times 7 = 2 \times \overline{v} = \overline{r}$ 

11. 
$$7 \times 6 = m \times 7 = 2 \times v = r$$

**12.** 
$$6 \times 8 = 3 \times 6 = 2 \times 2 = 2 = 2 = 2 \times 3 = 2 \times 3$$

13. 
$$9 \times 6 = x \times 9 = 2 \times z = a$$

Just For Fun

'lay "Buzz" with your neighbor.

You pick a number less than 10, for example 7. Then, taking turns, you and your neighbor start counting from 1. If you say a number which has 7 as a digit, or which is a product with 7 as a factor you say, "Buzz." If you fail to say, "Buzz," your neighbor scores a point. The winner of the game is the one with the most points when you reach 9 times the number you picked.

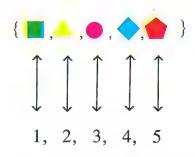
iee if you can see the rule for finding these missing iddends quickly.

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# CHAPTER 8 · Sets, Numbers, Numerals

### Finite sets

If the cardinal number of a set is a whole number, then we call the set a finite set. We usually find the cardinal number of a finite set by listing the objects so that they can be counted.



#### EXERCISES

The objects in each of these finite sets are listed for you. Name the cardinal number of each set.

2. 
$$\{10-2, \text{ eight}, 4 \times 2, 8, 16 \div 2, 6+2\}$$

The objects in each of these finite sets are described for you. Name the cardinal number of each set.

Name the cardinal numbers of these sets.

**9.** 
$$A = \{3, 5, 7, 9, 11, 13\}$$

**10.** 
$$S = \{7, 9, 11\}$$

11. 
$$B = \{\text{the children in your class}\}\$$

12. 
$$X = \{\text{the boys in your class}\}\$$

**13.** 
$$C = \{\text{the letters in the alphabet}\}\$$

**14.** 
$$R = \{u, v, w, x, y, z\}$$

**15.** 
$$M = \{ \text{the days of the week} \}$$

16. 
$$P = \{Monday, Sunday\}$$

**17.** 
$$N = \{ \text{whole numbers from 1 to 10} \}$$
 **18.**  $Q = \{3, 4, 9, 5, 2\}$ 

**18.** 
$$Q = \{3, 4, 9, 5, 2\}$$

**19.** 
$$O = \{ \text{the pages in this book} \}$$

**20.** 
$$R = \{ \text{page } 10, \text{page } 222 \}$$



If we cannot find a cardinal number for a set which is a whole number, then the set is an **infinite set**. The set  $\{0, 1, 2, 3, \ldots\}$  is infinite. Can you think of other infinite sets?

#### EXERCISES

he objects in each of these sets are **described** for you. Vhich of these sets are infinite?

- 1. {the names for ten}
- 3. {the states in the United States}
- **5.** {the letters in the alphabet}

- 2. {the sides of a triangle}
- **4.** {all the numbers greater than 100}
- **6.** {all the shapes that can be drawn}

- escribe each of these infinite sets.
  - **7.** {0, 2, 4, 6, 8, 10, 12, 14, 16, . . .}
- **8.** {1, 3, 5, 7, 9, 11, 13, ...}

nswer the questions.

- **9.** Do numbers like "first, second, third, fourth and so on" form an infinite set?
- **10.** Can you **list** all the members of an infinite set?
- 11. Can you name the cardinal number of the set {1, 2, 3, 4, 5, . . .}?

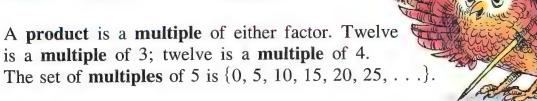
Bob looked at the set in question 11 and said, "Any number you can name in that set, I can name a greater one." Barry said, "Nine million." Bob said, "Nine million and one."

- 12. Is Bob's way a good way to test whether the set is infinite?
- **13.** Can you give a rule for naming a number that is greater than the one anyone else names?

### Multiples

 $3 \times 4 = 12$  product or factor factor multiple

THE I CLIPPLY TO GET A MINTHUME



Zero is a **multiple** of **any** counting number because the product of 0 and any number is 0.

#### EXERCISES

Write the numerals to complete these sets of multiples.

1. 
$$\{0, 2, 4, 6, 8, \mathbf{a}, \mathbf{r}, \mathbf{c}, \mathbf{n}, 18\}$$

**2.** 
$$\{0, 4, 8, 12, 16, \overline{a}, \overline{v}, \overline{u}, \overline{z}, 36\}$$

**3.** 
$$\{0, 8, 16, 24, 32, \mathbf{e}, \mathbf{r}, \mathbf{s}, \mathbf{x}, 72\}$$

5. 
$$\{0, 6, 12, 18, 24, 5, n, 0, r, 54\}$$

**6.** 
$$\{0, 9, 18, 27, 36, \overline{u}, \overline{v}, \overline{a}, \overline{x}, 81\}$$

Name these sums.

In exercises 1-6 you wrote the first ten multiples of

2, 3, 4, 6, 8, and 9. The next four multiples of 2 after

18 are 20, 22, 24, 26.

Write the next four multiples

Name the first five multiples of

### Multiples on the number line



70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90

On the number line we have marked 70 as a multiple of 7. To find the next multiple of 7 we move 7 to the right and stop at 77.

#### EXERCISES

Use the number line to help you name the multiples.

- 1. 72 is a multiple of 6. Name the next 2 multiples of 6.
- 2. 72 is a multiple of 8. Name the next 2 multiples of 8.
- **3.** 72 is a multiple of 9. Name the next 2 multiples of 9.
- 4. 80 is a multiple of 5. Name the next 2 multiples of 5.
- **5.** 81 is a multiple of 3. Name the next 2 multiples of 3.
- **6.** 90 is a multiple of 6. Name the 2 multiples of 6 just before 90.
- **7.** 90 is a multiple of 3. Name the 2 multiples of 3 just before 90.
- **8.** 88 is a multiple of 4. Name the 2 multiples of 4 just before 88.
- **9.** 70 is a multiple of 7. Name the multiple of 7 between 70 and 84.

Jse the number line to help you name the multiples.

# 101 102 103 104 105 106 107 108 109 110 171 112 113 114 115 116 117 118 119 120

- 10. 104 is a multiple of 4. Name the next 2 multiples of 4.
- 11. 105 is a multiple of 7. Name the next 2 multiples of 7.
- 12. 108 is a multiple of 2. Name the next 2 multiples of 2.
- 13. 105 is a multiple of 3. Name the next 2 multiples of 3.
- **14.** 120 is a multiple of 8. Name the 2 multiples of 8 just before 120.

### Odd and even numbers



Answer the questions.

- 1. 0, 2, 4, 6, 8 are even numbers. Name the next 6 even numbers.
- 2. The compact numerals for even numbers always end in  $\overline{a}$ ,  $\overline{n}$ ,  $\overline{c}$ ,  $\overline{x}$ , or  $\overline{e}$ .
- **3.** 1, 3, 5, 7 are odd numbers. Name the next 6 odd numbers.
- 4. The compact numerals for odd numbers always end in  $\overline{a}$ ,  $\overline{c}$ ,  $\overline{e}$ ,  $\overline{n}$ , or  $\overline{o}$ .
- 5. Name the 6 even numbers between a. 15 and 27 b. 81 and 93.
- 6. Name the 6 odd numbers betweena. 20 and 32b. 90 and 102.
- 7. Name the next 5 even numbers after **a.** 17 **b.** 94 **c.** 324 **d.** 475
- 8. Name the next 5 odd numbers after g. 13 b. 46 c. 284 d. 300 e. 7248
- 9. Name the 5 even numbers just beforea. 100b. 43c. 400d. 2564
- 10. Name the 5 odd numbers just before a. 100 b. 42 c. 398 d. 2564 e. 5298

Complete the equations.

**11.** 
$$11 = (\overline{a} \times 2) + 1$$
 **12.**  $15 = (\overline{r} \times 2) + 1$  **13.**  $21 = (\overline{c} \times 2) + 1$ 

**14.** 
$$19 = (\overline{S} \times 2) + 1$$
 **15.**  $41 = (\overline{e} \times 2) + 1$  **16.**  $61 = (\overline{z} \times 2) + 1$ 

Answer the questions.

17. Is 
$$6 \times 2$$
 even?

18. Are even numbers multiples of 2?

**19.** Is 
$$(6 \times 2) + 1$$
 odd?

**20.** Are odd numbers always 1 more than multiples of 2?

### Odd and even numbers



Name the sums or missing addends.

When you add or subtract two odd numbers, the sum or missing addend is always an \_\_? number.

lame the sums or missing addends.

When you add or subtract two even numbers, the sum or missing addend is always an \_\_\_? number.

lame the sums or missing addends.

When you add or subtract an odd number and an even number the sum or the missing addend is always an ? number.

ame the products.

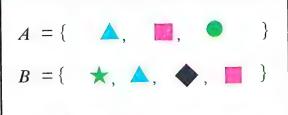
$$\times 2$$

When one of the factors is even, the product is always ? . If the product is even, one of the factors is always \_\_\_?\_\_.

ame the products.

When both factors are odd, the product is always \_\_?\_. If the product is odd, both factors are always \_\_?\_.

### Common multiples



and are in both sets.

They are common to both sets.

 $A = \{0, 2, 4, 6, 8, 10, 12, 16, 18\}$ Set A contains multiples of 2.  $B = \{0, 3, 6, 9, 12, 18\}$ Set B contains multiples of 3. 0, 6, 12, 18 are in both sets of multiples.

They are **common multiples** of 2 and 3.

#### EXERCISES

Name the objects that are common to each pair of sets.

- 1.  $\{a, b, c, d, e\}$
- 2. {a fly, a carrot, a beetle}
- 3. {John, Dick, Harry, George, Eileen, Sue}

 $\{m, c, b, n\}$ 

{a beetle, a cabbage}

{Sue, Harry, Gay, Eileen, Fran, Joe}

Answer the questions.

Here are the multiples of 4 and 5 to 40:

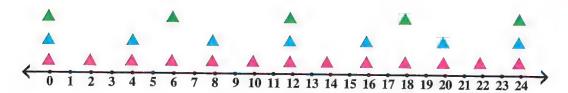
- $\{0, 4, 8, 16, 20, 24, 28, 32, 36, 40\}$
- $\{0, 5, 10, 15, 20, 25, 30, 35, 40\}$ 
  - 4. Name the multiples that are common to both sets.
  - 5. If the sets were continued, what would be the next common multiple?

Here are the multiples of 6 and 9 to 60:

- $\{0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60\}$
- $\{0, 9, 18, 27, 36, 45, 54\}$ 
  - 6. Name the common multiples of 6 and 9 less than 60.
  - 7. Name the next common multiple of 6 and 9 after 54.
  - 8. Name the common multiples of 4 and 6 less than 40.
  - **9.** List the multiples of 3 and 4 to 36.
- 10. Name the common multiples of 3 and 4 less than 36.

### Common multiples on the number line



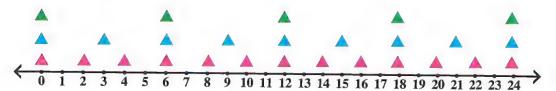


We have marked the multiples of 2 with red, the multiples of 4 with blue, the multiples of 6 with green. The set of multiples of 2 on the number line is  $\{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24\}$ . The set of multiples of 6 is  $\{0, 6, 12, 18, 24\}$ . There are 5 multiples that are common to the set for 2 and the set for 6. They are  $\{0, 6, 12, 18, 24\}$ .

#### EXERCISES

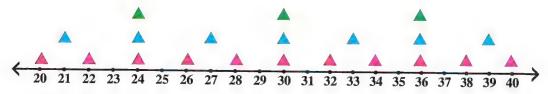
- 1. Use the number line above to name the set of common multiples from 0 to 24 of
  - **a.** 2 and 4

**b.** 4 and 6



- 2. Use the number line above to name the set of common multiples from 0 to 24 of
  - **a.** 2 and 3

**b.** 3 and 6



- **3.** Use the number line above to name the set of common multiples from 20 to 40 of
  - **a.** 2 and 3

**b.** 3 and 6

**c.** 2 and 6

$$12 \times 1 = 12$$

$$3 \times 4 = 12$$

$$2 \times 6 = 12$$

In each of the equations two factors are renamed as a product. The products in the equations are the same, but the factors are different. There are three pairs of factors whose products are 12. We can write the factors of 12 as {1, 2, 3, 4, 6, 12}.

#### EXERCISES

Write the equations to show that

- 1. 2 and 3 are factors of 6
- 3. 3 and 6 are factors of 18
- 2. 3 and 4 are factors of 12
- 4. 8 and 9 are factors of 72

Study these equations. Then list the factors of 8, 6,

9, 10, and 24 in set form.

5. 
$$2 \times 4 = 8$$
 and  $8 \times 1 = 8$ 

**6.** 
$$2 \times 3 = 6$$
 and  $6 \times 1 = 6$ 

7. 
$$3 \times 3 = 9$$
 and  $9 \times 1 = 9$ 

**8.** 
$$2 \times 5 = 10$$
 and  $10 \times 1 = 10$ 

**9.** 
$$3 \times 8 = 24$$
 and  $6 \times 4 = 24$ 

10. 
$$12 \times 2 = 24$$
 and  $24 \times 1 = 24$ 

Write the numerals to complete this table.

11.	factor	3	6	3	2	4	4	6
	factor	_4	-4	6	e	_ <b>n</b>	a_	r
	product	12	s	C.	10	16	32	24

Solve these equations.

12. 
$$3 \times [a] = 30$$

13. 
$$6 \times v = 30$$

14. 
$$2 \times \boxed{c} = 30$$

15. 
$$1 \times r = 30$$

16. 
$$18 \times e = 36$$

17. 
$$4 \times \mathbf{u} = 36$$

18. 
$$12 \times s = 36$$

19. 
$$6 \times \mathbf{z} = 36$$

**20.** 
$$6 \times \bar{n} = 42$$

**21.** 
$$2 \times [0] = 42$$

**22.** 
$$3 \times x = 42$$

**23.** 
$$1 \times s = 42$$

From the equations above write

- 24. a set of eight factors of 30
- 25. a set of seven factors of 36
- 26. a set of eight factors of 42

#### **Factors**



1 set of 12  
1 
$$\times$$
 12 = 12

$$2 \text{ sets of } 6$$
$$2 \times 6 = 12$$

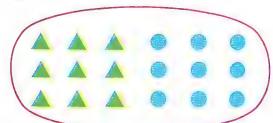
$$3 \text{ sets of } 4$$
$$3 \times 4 = 12$$

The factors of 12 are 1, 12, 2, 6, 3, 4.

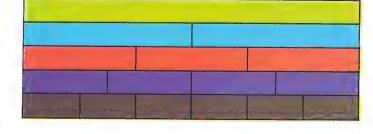
### EXERCISES

olve the equations.

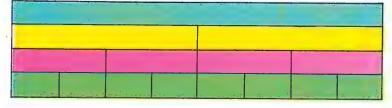
- 1. 1, 18, 2, 9, 3, 6 are <u>?</u> of 18.
- **2.**  $1 \times a = 18$
- 3.  $2 \times c = 18$
- 4.  $3 \times e = 18$
- 5.  $6 \times \bar{n} = 18$
- 6.  $9 \times |r| = 18$



- 7. The set of factors of 36 is  $\{a, c, e, n, z,$
- [r, s, u, x]8.  $36 \times [a] = 36$
- 9.  $18 \times \boxed{c} = 36$
- 10.  $12 \times [e] = 36$
- 11.  $9 \times [n] = 36$
- 12.  $6 \times [r] = 36$



- 13. The set of factors of 40 is  $\{a, c, e, n, x, r, s, u\}$
- 14.  $40 \times a = 40$
- 15.  $20 \times [c] = 40$
- **16.**  $10 \times [e] = 40$
- 17.  $5 \times [n] = 40$



omplete the sentence.

18. a is a factor of every counting number.

### Prime numbers

$$1 \times 2 = 2$$

$$1 \times 3 = 3$$

$$1 \times 5 = 5$$
  
 $1 \times 13 = 13$ 

$$1 \times 7 = 7$$

$$1 \times 11 = 11$$

2, 3, 5, 7, and 11 each have only two factors. Every counting number greater than 1 that has only two factors is called a prime number.

### EXERCISES

### Copy this chart.

	/								
	2	3	4	5	6	7	8	9	10
						17			
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
						47			
51	52	53	54	55	56	57	58	59	60

- 1. On your chart strike out the multiples
  - a. of 2, greater than 2
  - **b.** of 3, greater than 3
  - c. of 5, greater than 5 **d.** of 7, greater than 7
- 2. How many multiples are there
  - a. of 2, from 2 to 60
  - c. of 5, from 5 to 60
- **b.** of 3, from 3 to 60
- **d.** of 7, from 7 to 60

### Using the chart, can you name any factors

- **3.** for 13, except 1 and 13?
- **5.** for 19, except 1 and 19?
- **7.** for 29, except 1 and 29?
- 9. for 53, except 1- and 53?

- **4.** for 17, except 1 and 17?
- **6.** for 23, except 1 and 23?
- **8.** for 43, except 1 and 43?
- 10. for 59, except 1 and 59?

### Answer the questions.

- 11. Which of these numbers are prime numbers?
  - **a.** 9 **b.** 10
- c. 11 d. 27 e. 28 f. 29
- **g.** 49
- **h.** 59

h. 55

- 12. Name the first prime number after
  - **a.** 10 **b.** 20 **c.** 30 **d.** 35 **e.** 40 **f.** 45

- **g.** 50
- 13. Can a prime number greater than 2 be an even number? Explain.
- 14. List the counting numbers from 2 to 60 that are not prime numbers because they are multiples of 2, 3, 5 or 7.

### More on prime numbers



Ise this number pattern to help you. We have ringed ne prime numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31)	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

- 1. How many prime numbers are less than 10?
- 2. How many prime numbers are less than 20?
- 3. How many prime numbers are less than 50?
- 4. Name four prime numbers between 18 and 34.

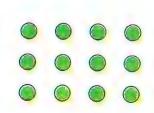
Copy this number chart.

1	7	13	19	25	31	37	43	49	55
2	8	14	20	26	32	38	44	50	56
3	9	15	21	27	33	39	45	51	57
4	10	16	22	28	34	40	46	52	58
5	11	17	23	29	35	41	47	53	59
6	12	18	24	30	36	42	48	54	60

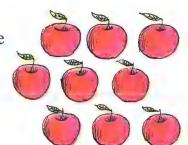
- 5. On your chart ring the prime numbers.
- 6. Complete this set of multiples of 6 {0, 6, 12, 18, **a**, **c**, **e**, **n**, **r**, **s**, 60}.
- 7. What can you say about prime numbers from the chart, using the set of multiples of 6?
- 8. Is 1 a prime number?
- **9.** Is 0 a prime number?
- 10. What is the only even prime number?

### Factors and prime numbers

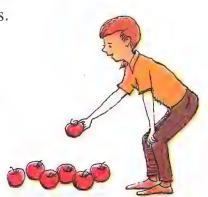
1. Our picture shows 12 dots arranged to form a rectangular pattern with 3 rows of 4 dots. Can you arrange the dots to form another rectangular pattern? If so, how many rows? How many dots in each row?



- **2. a.** Can you arrange 17 dots to form a rectangular pattern?
  - **b.** How can you arrange the dots into a rectangular pattern with the least number left over?
- **3. a.** Can you arrange 23 dots to form a rectangular pattern?
  - **b.** Why?
- 4. Bob had 9 apples. He wanted to arrange them into 2 or more sets, each containing the same number of apples. How many ways could he do this?



- 5. Jon had 7 apples. He wanted to arrange them into 2 or more sets, each containing the same number of apples. Could he do this?
- **6.** Jon ate one of his apples. Could he arrange the apples he had left into groups with the same number of apples? In how many ways?



234 (two hundred thirty-four)

### Using sets and numbers



. If you had 24 tickets to sell for the school concert and had sold 15, how many would you have left to sell?



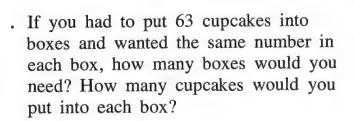
- . If 7 children each sold 6 tickets, how many tickets in all were sold?
- There are 50 pieces in Richard's jigsaw puzzle, and he has fitted in 36. How many does he still have to do?

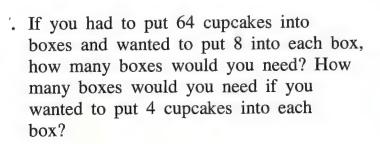


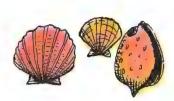
. If you were putting plums into jars and knew that 8 plums fitted into a jar, how many jars would you need for 72 plums?



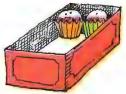
David had 17 shells, and Carol had 13. David gave Carol 2 shells. How many shells did David have then? How many shells did Carol have?



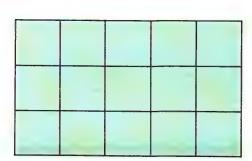








### How many squares?



There are 15 small square regions in this rectangular region. If the small square region is the unit of area, there are 3 units on one side and 5 units on the other side.

$$3 \times 5 = 15$$
  
m (region) = 15

The area is 15 small squares.

### EXERCISES

Complete the equations.



1. 
$$2 \times 8 = \mathbf{a}$$
  
m (region) =  $\mathbf{n}$ 

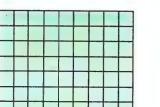
2. 
$$8 \times 2 = \boxed{c}$$
  
m (region) =  $\boxed{e}$ 

4.  $7 \times s = 28$ 

m (region) = u



3. 
$$4 \times 7 = \mathbf{e}$$
  
m (region) =  $\mathbf{r}$ 



5. 
$$8 \times 9 = \overline{v}$$
  
m (region) =  $\overline{z}$ 

6. 
$$9 \times \overline{r} = 72$$
  
m (region) =  $\overline{c}$ 

236 (two hundred thirty-six)



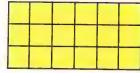
	1111	111 11	11 "	Area
100 10 10 10 10 10 10 10 10 10 10 10 10	11 /11 /11	1 11	11 1	
	111 111	1/1 //	1 111	
	1/11 1/1 1/1 1/1	-		
		111 1	111 1111111	

George helped his father put tile on the wall. He said, "There are 9 tiles across the bottom and 7 tiles up one side. This part of the wall will take 63 tiles." Was George right?

### EXERCISES

Name the number of units of area.







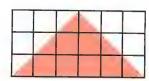
1. m (region) = a

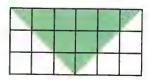
2. m (region) = 0

3. m (region) =  $\overline{c}$ 

lame the number of units of area.







4. m (region) =  $\overline{a}$  5. m (region) =  $\overline{r}$ 

6. m (region) = e

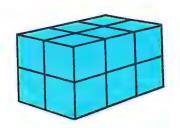
olve the problems.

- 7. The unit of area for measuring this piece of cloth is a square inch. Beryl knows that there are 72 square inches in the cloth and that one side of the cloth is 8 inches. How long is the other side?
- 8. Beth is making a quilt from square pieces of cloth. She fits 8 pieces along one side and 12 pieces along the other. How many pieces of cloth will she need?





### Volume



There are 12 small boxes in the larger box. If the small box is the **unit of volume** there are 2 units on one side, 3 units on the second side, and 2 units on the third side.

$$2 \times 3 \times 2 = 12$$
  
m (box) = 12

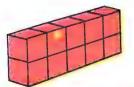
The volume is 12 small boxes.

### EXERCISES

Name the number of little boxes that will fit in the larger boxes.



1. 
$$m \text{ (box)} = a$$



3. 
$$m (box) = c$$

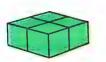


5. 
$$m (box) = e$$

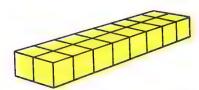
Copy and complete these equations.

7. 
$$2 \times 3 \times 3 = (2 \times 3) \times 3$$
  
=  $\boxed{s} \times 3$   
=  $\boxed{o}$ 

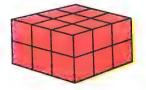
**9.** 
$$(4 \times 3) \times 2 = 4 \times (3 \times \mathbf{z}) = \mathbf{u}$$



**2.** m (box) = 
$$n$$



**4.** m (box) = 
$$o$$



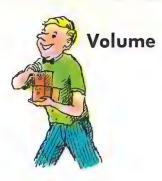
6. m (box) = 
$$r$$

8. 
$$2 \times 3 \times 3 = 2 \times (3 \times 3)$$
  
=  $2 \times \boxed{e}$   
=  $\boxed{x}$ 

10. 
$$5 \times (2 \times 3) = T$$

238 (two hundred thirty-eight)

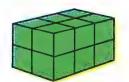




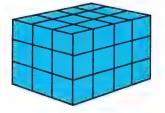
Jane was helping Mr. Williams with his box of milk cartons. She asked, "How many cartons of milk will your box hold?" He said, "Well, the box is 3 cartons wide, 4 cartons long, and 2 cartons high. Can you tell me?" "Sure," said Jane, " $3 \times 4 \times 2 = 24$ , so you can get 24 cartons in the box." Was Jane right?

#### EXERCISES

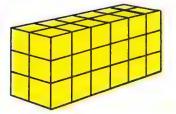
lame the number of units of volume of each box.



1. 
$$m (box) = a$$



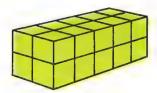
3. 
$$m (box) = \overline{c}$$



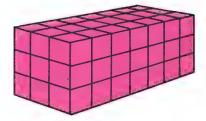
5. 
$$m (box) = e$$



**2.** m (box) = 
$$r$$



4. 
$$m (box) = u$$



6. 
$$m (box) = n$$

(two hundred thirty-nine) 239

## Tens and hundreds

Complete these equations.

1. 
$$3 \times 20 = 3 \times (2 \times 10) = (3 \times \mathbf{a}) \times 10 = \mathbf{c} \times 10 = \mathbf{e}$$

**2.** 
$$4 \times 40 = 4 \times (4 \times 10) = (4 \times \mathbf{a}) \times 10 = \mathbf{c} \times 10 = \mathbf{e}$$

3. 
$$7 \times 40 = 7 \times (4 \times 10) = (7 \times \mathbf{a}) \times 10 = \mathbf{c} \times 10 = \mathbf{e}$$

**4.** 
$$8 \times 60 = 8 \times (6 \times 10) = (8 \times 2) \times 10 = 2 \times 10 = 2$$

**5.** 
$$3 \times 200 = 3 \times (2 \times 100) = (3 \times \boxed{a}) \times 100 = \boxed{c} \times 100 = \boxed{e}$$

**6.** 
$$4 \times 400 = 4 \times (4 \times 100) = (4 \times \mathbf{a}) \times 100 = \mathbf{c} \times 100 = \mathbf{e}$$

**7.** 
$$7 \times 400 = 7 \times (4 \times 100) = (7 \times \mathbf{a}) \times 100 = \mathbf{c} \times 100 = \mathbf{e}$$

**8.** 
$$8 \times 600 = 8 \times (6 \times 100) = (8 \times \boxed{a}) \times 100 = \boxed{c} \times 100 = \boxed{e}$$

**9.** 
$$20 \times 10 = (2 \times 10) \times 10 = 2 \times (10 \times 10) = \overline{a}$$

**10.** 
$$20 \times 40 = (2 \times 10) \times (4 \times 10) = (2 \times 4) \times (10 \times 10) = \boxed{c}$$

11. 
$$70 \times 40 = (7 \times 10) \times (4 \times 10) = (7 \times 4) \times (10 \times 10) = \boxed{e}$$

**12.** 
$$80 \times 50 = (8 \times 10) \times (5 \times 10) = (8 \times 5) \times (10 \times 10) = \mathbf{n}$$

How many tens are there in

How many hundreds are there in

Solve these equations.

**21.** 
$$3 \times 40 = a$$

**21.** 
$$3 \times 40 = \overline{a}$$
 **22.**  $6 \times 90 = \overline{c}$ 

**23.** 
$$8 \times 50 = [e]$$

**24.** 
$$3 \times 400 = n$$

**25.** 
$$90 \times 600 = r$$

**26.** 
$$8 \times 500 = s$$

Name the tens' digit in

Name the hundreds' digit in

Write 3-digit numerals for these numerals.

**39.** 
$$100 + 20 + 4$$

**43.** 
$$600 + 30 + 9$$

**44.** 
$$500 + 3$$

### Thousands

Write these numerals in short form.

1. 
$$4000 + 200 + 50 + 7$$

3. 
$$8000 + 500 + 20 + 4$$

Write in short form.

**7.** 
$$3 \times 10$$
 **8.**  $30 \times 10$  **9.**  $3 \times 100$  **10.**  $30 \times 100$ 

11. 
$$21 \times 10$$

**11.** 
$$21 \times 10$$
 **12.**  $21 \times 100$ 

**13.** 
$$39 \times 100$$
 **14.**  $40 \times 100$ 

14. 
$$40 \times 100$$

Name the thousands' digit in

Name the tens' digit in

Write in words.

Write in short form.

- 31. four thousand, six hundred twenty-seven
- 32. seven thousand, six hundred ninety-six
- 33. eight thousand, four hundred
- 34. eight thousand, forty

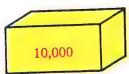
Write numerals for

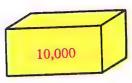
Write the numeral to replace each  $\Box$ .

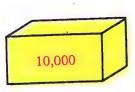
- 38. The numeral for this year is a.
- **39.** The numeral for the year you were born is  $\overline{c}$ .
- **40.** There are **@** minutes in 10 hours.
- 41. There are n weeks in 100 years if each year has 52 weeks.

/rite 6-digit numerals for

## Tens of thousands







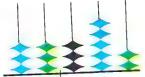
Study the boxes above, then answer these questions.

- 1. Each of these boxes contains ten thousand boxes. How many thousands are there in all? How would you write "thirty thousand" with digits?
- 2. How many thousands are there in
  - **a.** 19,000
- **b.** 23,000
- **c.** 46,258
- **d.** 76,489 **e.** 93,117

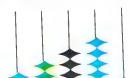
- 3. Write 5-digit numerals for
  - a. twenty-seven thousand, six hundred forty-five
  - b. forty-eight thousand, three hundred forty-five
- 4. If you had an abacus to show tens of thousands, how many rods would you need?

Write 5-digit numerals for the numbers shown here.

5.



6.



7.



Look at the numeral 34,486. We separate the numeral into two parts by writing a comma after the thousands' place. The first part, thirty-four, is thousands; the second part, four hundred eighty-six, is ones. The numeral may be read as "thirty-four thousand, four hundred eighty-six ones."

Write 5-digit numerals for

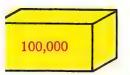
	Thousands	Ones
8.	43	678
0.	29	600

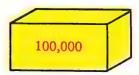
	Thousands	Ones
9.	16	726
11.	97	7

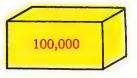
Write 5-digit numerals for

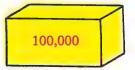
### Hundreds of thousands











tudy the boxes above, then answer these questions.

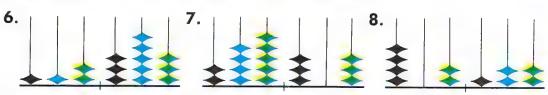
- 1. Each of these boxes contains one hundred thousand boxes. How many thousands are there in all? How would you write "four hundred thousand" with digits?
- **2.** Count by hundreds of thousands from 100,000 to 900,000. Write the multiples as you count 100,000, 200,000, 300,000, . . .
- 3. How many thousands are there in
  - a. 219,000

**b.** 289,000

c. 476,241

- 4. Write 6-digit numerals for
  - a. 246 thousand
  - b. three hundred forty-six thousand
  - c. two hundred seventy thousand
  - d. 892 thousand
- 5. If you had an abacus to show hundreds of thousands, how many rods would you need?

rite 6-digit numerals for the numbers shown here.

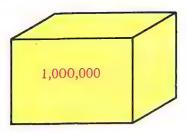


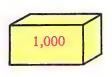
rite 6-digit numerals for

	Thousands	Ones	
9.	243	289	
11.	382	645	

	Thousands	Ones
10.	823	467
12.	356	68

rite 6-digit numerals for





The box on the right contains a thousand ones. The box on the left contains a thousand thousands. Another name for a thousand thousands is a **million**.

We read 6,483,276 as, "Six million, four hundred eighty-three thousand, two hundred seventy-six."

#### EXERCISES

For each of these problems name the number of millions, the number of thousands, and the number of ones.

1. 4,683,297

**2.** 3,764,281

**3.** 9,480,274

**4.** 7,293,400

**5**. 6,800,270

**6.** 5,031,068

#### Write the numerals for

	Millions	Thousands	Ones
7.	7	486	291
8.	7	300	486
9.	2	390	273
10.	8	481	50

### Answer the questions.

- 11. Name the next thousand after 3,486,976; after 4,689,273; after 13,420,386.
- **12.** Name the next million after each number in question 11.
- 13. Write another name for two thousand thousands; for twenty hundred thousands; for ninety hundred thousands.

#### 244 (two hundred forty-four)

#### Millions



Read these facts. See if you can find other interesting number facts with millions. Write them out and put them on the bulletin board.

- 1. The number of times you breathe in a year is about 8,935,000.
- **2.** There were 7,781,984 people living in New York City in 1960. How many people live in your town?
- **3.** The area of the United States is 3,628,510 square miles.
- **4.** The world record for an attack of hiccoughs is for a period of 8 years and 160,000,000 hiccoughs.
- **5.** The total attendance at the major league baseball games in 1964 was 21,280,346. Did you go to a game?
- 6. The earth is about 92,900,000 miles from the sun. You can imagine how hot it must be on the sun for us to feel the warmth this far away on earth.
- **7.** In 1965, there were 61,500,000 television sets in the United States.

#### EXTRA FOR EXPERTS



ame the sums.

ame the missing addends.



- 1. Mary went with her mother to buy clothes for a vacation. She spent these amounts: blouse \$4.20, coat \$17.50, shoes \$8.50, skirt \$8.00.
  - a. How much did Mary spend?
  - **b.** How much more did she spend for shoes than for the skirt?
- 2. Eddie went with his mother to buy some clothes. He spent these amounts: shirt \$4.00, sweater \$7.50, shoes \$8.00, jacket \$15.00.
  - a. How much did Eddie spend?
  - **b.** How much more did he spend for shoes than for the sweater?

Name the sums. Remember that the value of \$1 is 100¢.

5. 
$$\$71 + 18$$

Name the missing addends.

9. 
$$$76$$
 $-28$ 
 $\frac{}{\$}$ 

12. 
$$$47$$
 $-26$ 
 $\frac{}{\$}$ 

17. 
$$\$72$$
 $\frac{-50}{\$}$ 

$$\frac{$}{$}$$
 19.  $\frac{62}{$}$   $\frac{-19}{$}$ 



1. Judy wanted to add the prices. The list was too long for her to add all at once, so she cut it into 5 pieces like this.

·····				
.23	.41	2.64	1.19	.65
.31	.26	.31	.14	.65
.31	.26	.27	.31	.07
.19	.11	.15	.31	.23

hen Judy found the sum of each column. She added ese 5 sums together to get the total amount.

Judy had \$10, how much change should she get?

2. Find the totals for the 6 separate pieces of tape, then find the total for the six together.

~~~~~~					
.21	.16	.15	.33	.63	.30
.31	.23	. 17	.33	.36	.14
.40	.23	1.18	.45	.27	.14
.17	.41	.56	.47	.27	.14

3. Find the totals for the 6 separate pieces of tape, then find the total of the six together.

~~~~~~					
.23	.17	2.34	.26	.53	.67
.31	. 18	.14	.26	.53	.61
.41	.26	.56	.26	.24	.89
.17	.47	.28	.11	.19	.99
					h

(two hundred forty-seven) 247

## Practice page

Name the sums.

Name the total amounts.

7. 
$$23 ¢ 8. 63 ¢ 9. $48 10. $53 11. 39 ¢ 12. 73 ¢ 
 $+48 ¢ ¢ +29 ¢ $ +57 ¢ $ +68 ¢ $ +54 ¢ $ +87$$$

Name the missing addends.

Name the missing amounts.

19. 
$$71 \stackrel{\circ}{\leftarrow}$$
 20.  $33 \stackrel{\circ}{\leftarrow}$  21.  $56 \stackrel{\circ}{\leftarrow}$  22.  $81 \stackrel{\circ}{\leftarrow}$  23.  $$17$  24.  $$34$   $-28$   $\stackrel{\circ}{\leftarrow}$   $\stackrel{\circ}{\leftarrow}$   $\frac{-14}{\stackrel{\circ}{\leftarrow}}$   $\frac{-38}{\stackrel{\circ}{\leftarrow}}$   $\frac{-27}{\stackrel{\circ}{\leftarrow}}$   $\frac{-9}{\stackrel{\circ}{\Rightarrow}}$   $\frac{-16}{\stackrel{\circ}{\Rightarrow}}$ 

Solve the problems.

- 25. What is the total cost of a hat at \$4, a coat at \$17, a jacket at \$11, and shoes at \$5?
- How much more would it cost to buy 2 hats at \$4 each than to buy 3 shirts at \$2 each?
- One dozen eggs costs 43¢. How much do 3 dozen cost?
- 28. One pint of orange juice costs 21¢. How much for 4 pints?
- 29. How much for 4 cans of fruit juice if one costs
- **30**. How much for 1 can of tomatoes if 3 cans cost 99¢?



- 1. Write the multiples of **a**. 7, to 42 **b**. 9, to 63.
- 2. Write the factors of a. 8 b. 15.
- 3. Can you name the cardinal number of this set with a whole number: {the ideas that people can have}? What kind of set is this?
- **4.** Name the prime numbers in this set: {2, 4, 7, 8, 10, 11, 12, 13}
- **5.** Name the odd numbers in this set: {3, 17, 72, 3841, 57, 8336}
- 7. Name the tens' digit in **a.** 365 **b.** 4768 **c.** 62,345 **d.** 786,294
- 8. Name the thousands' digit ina. 4683b. 30,964c. 289,364d. 9,998,999
- Write numerals for a. two hundred twenty-seven thousand, four hundred forty b. three hundred fifty thousand, six hundred eighty-nine.
- 10. Name the number 10 greater thana. 4368b. 2998c. 31,478
- 11. Name the number 100 greater than a. 4682 b. 5928 c. 23,199
- **12.** Name the number 1000 greater than **a.** 4682 **b.** 15,928 **c.** 29,384
- **13.** Name the number 100,000 greater than **a.** 300,000 **b.** 486,597 **c.** 4
- 14. How many digits are used in writing the common numeral for a million?

A product of two numbers is a **multiple** of each number. For instance, 12 is a multiple of 3 and of 4.

- 1. Write the multiples of 4 which are less than 30.
- 2. How many multiples of 6 are less than 50?

When two numbers are multiplied to give a product, the numbers are **factors** of the product. For instance, 3 and 4 are factors of 12.

- 3. Write two factors of 10 other than 10 and 1.
- 4. Write four factors of 12 other than 4 and 3.

If an object belongs in two sets, it is said to be **common** to the two sets. For instance, 2 and 31 are common to {2, 4, 9, 16, 31} and {3, 8, 11, 2, 31}.

- 5. Name the common multiples. multiples of 3: {3, 6, 9, 12, 15, 18} multiples of 2: {2, 4, 6, 8, 10, 12, 14, 16, 18}
- **6.**—Name the common factors. factors of 12: {1, 2, 3, 4, 6, 12} factors of 18: {1, 2, 3, 6, 9, 18}

Any number, greater than one, which has only two factors is called a **prime number**. For instance, 7 has only two factors, 7 and 1. 7 is a prime number.

- 7. Name the prime numbers in these sets. {11, 12, 13, 14} {50, 51, 52, 53}
- **8.** Why is it that every prime number greater than 2 must be odd?

#### EXTRA FOR EXPERTS

Many years ago the Romans used numerals ifferent from the ones we use. Roman numerals re still used in some places. Some clocks use these ymbols. Both of the clocks on the right use Roman umerals.

To name the numbers up to 30, the Romans used iree symbols, I, V, and X. The I stood for 1, the for 5 and the X for 10. These symbols were pined together to show numbers like this.





$$= I$$
,  $2 = 1 + 1 = II$ ,  $3 = 1 + 1 + 1 = III$   $4 = 5 - 1 = IV$ 

Γo show that the I is subtracted, it was written to the ft of the V.)

#### EXERCISES

rite the Roman numerals for

1. 
$$6 = 5 + 1 = a$$

3. 
$$8 = 5 + 1 + 1 + 1 = \overline{c}$$

5. 
$$11 = 10 + 1 = e$$

**2.** 
$$7 = 5 + 1 + 1 = \boxed{n}$$

**4.** 
$$9 = 10 - 1 = 0$$

**6.** 
$$12 = 10 + 1 + 1 = \boxed{s}$$

ere are some equations written in Roman numerals. 'rite these equations in our kind of numerals.

$$7. I + V = VI$$

7. 
$$I + V = VI$$
 8.  $V + V = X$ 

13. 
$$VI + VI = [\overline{a}]$$
 14.  $II + IX = [\overline{c}]$ 

14. II + IX = 
$$\overline{c}$$

16. VII + IV = 
$$[r]$$
 17. X - III =  $[s]$ 

17. 
$$X - III = [s]$$

ame the time shown on these clocks.

19.



20.



9. VI + IV = X

12. XII - IV = VIII

15. XI - V = [e]

18. X - V = n

21.



(two hundred fifty-one) 251

# CHAPTER 9 • Multiplication and Division



#### The common factor rule

Study these equations.

$$(5 \times 8) + (2 \times 8) = (5 + 2) \times 8$$
  
=  $7 \times 8$   
=  $56$   
 $(5 \times 8) + (2 \times 8) = 40 + 16$   
=  $56$ 

The addends  $(5 \times 8)$  and  $(2 \times 8)$  have a common factor. We can add the other factors 5 + 2 = 7 and multiply by the common factor  $7 \times 8 = 56$ , or we can multiply  $5 \times 8 = 40$ ,  $2 \times 8 = 16$  and add 40 + 16 = 56. This is the common factor rule.

#### EXERCISES

Solve these equations.

1. 
$$(5 \times 4) + (2 \times 4) = (5 + 2) \times 4$$
  
=  $\boxed{a} \times 4$   
=  $\boxed{r}$ 

2. 
$$(5 \times 4) + (2 \times 4) = 20 + \overline{c}$$
  
=  $\overline{u}$ 

3. 
$$(6 \times 9) + (2 \times 9) = (6 + 2) \times 9$$
  
=  $\boxed{e} \times 9$   
=  $\boxed{n}$   
4.  $(6 \times 9) + (2 \times 9) = \boxed{c} + 18$   
=  $\boxed{a}$ 

**4.** 
$$(6 \times 9) + (2 \times 9) = c + 1 = a$$

5. 
$$(5 \times 7) + (4 \times 7) = (5 + 4) \times 7$$
  
=  $\boxed{s} \times 7$   
=  $\boxed{z}$   
6.  $(5 \times 7) + (4 \times 7) = \boxed{v} + 28$   
=  $\boxed{e}$ 

6. 
$$(5 \times 7) + (4 \times 7) = \overline{v} + 28$$
  
=  $\overline{e}$ 

Solve these equations.

7. 
$$7 \times 4 = (4 + 3) \times 4 = (4 \times 4) + (3 \times 4) = \mathbf{a} + 12 = \mathbf{r}$$

**8.** 
$$9 \times 3 = (5 + 4) \times 3 = (5 \times 3) + (4 \times 3) = \mathbf{c} + 12 = \mathbf{s}$$

**9.** 
$$6 \times 7 = (4 + 2) \times 7 = (4 \times 7) + (2 \times 7) = \mathbf{e} + 14 = \mathbf{u}$$

**10.** 
$$4 \times 7 = 4 \times (5 + 2) = (4 \times 5) + (4 \times 2) = \mathbf{a} + 8 = \mathbf{n}$$

11. 
$$5 \times 8 = 5 \times (6 + 2) = (5 \times 6) + (5 \times 2) = \overline{C} + 10 = \overline{O}$$

252 (two hundred fifty-two)

### Common factor rule



Study the equations and the number patterns.

,		,	= 30
+ (4	×	<b>10</b> )	= 40
7	×	10	= 70

×	10
3	30
4	40
7	70

The common factor is 10.

$$\frac{(5 \times 8) = 40}{+(2 \times 8)} = \frac{16}{56}$$

X	8
5	40
2	16
7	56

The common factor is 8.

#### EXERCISES

Write the numerals to complete these equations.

1. 
$$(3 \times 7) = \mathbf{n}$$

$$+ (6 \times 7) = \mathbf{0}$$

$$9 \times 7 = \mathbf{r}$$

$$\begin{array}{cccc}
(3 \times 7) &= & \overline{n} \\
+ & (6 \times 7) &= & \overline{o} \\
\hline
9 \times 7 &= & \overline{r}
\end{array}$$
2.  $(3 \times 8) = \overline{s} \\
+ & (5 \times 8) = \overline{u} \\
\hline
8 \times 8 &= & \overline{s}$ 

3. 
$$(4 \times 7) = \boxed{a}$$
  
  $+ (2 \times 7) = \boxed{c}$   
  $\boxed{6 \times 7} = \boxed{e}$ 

4. 
$$(10 \times 4) = \boxed{a}$$
  
 $+ (2 \times 4) = \boxed{c}$   
 $12 \times 4 = \boxed{e}$ 

5.  $(10 \times 6) = \boxed{n}$   
 $+ (4 \times 6) = \boxed{e}$ 
 $+ (6 \times 7) = \boxed{c}$   
 $14 \times 6 = \boxed{r}$ 

6.  $(20 \times 7) = \boxed{a}$   
 $+ (6 \times 7) = \boxed{c}$   
 $26 \times 7 = \boxed{e}$ 

5. 
$$(10 \times 6) = \boxed{n}$$

$$+ (4 \times 6) = \boxed{e}$$

$$14 \times 6 = \boxed{r}$$

6. 
$$(20 \times 7) = \overline{a}$$
  
 $+ (6 \times 7) = \overline{c}$   
 $\overline{26 \times 7} = \overline{e}$ 

Write the numerals to complete these number patterns.

#### The common factor rule

Study the examples and the number patterns.

#### EXERCISES

30

Write the numerals to complete these equations.

1. 
$$3 + 4 = 7$$
  
 $\times 2 \times 2$   
 $\boxed{a + n} = \boxed{c}$ 

2. 
$$3 + 5 = 8$$
  
 $\times 6$   
 $\boxed{a + e} = \boxed{s}$ 

3. 
$$3 + 6 = 9$$
  
 $\times 4$   
 $\boxed{r + u} = \boxed{v}$ 

14  $\times 3$ 

4. 
$$8 = 6 + 2$$

$$\times 6 \qquad \times 6$$

$$\overline{x} = \overline{z} + \overline{a}$$

5. 
$$9 = 5 + 4$$

$$\times 7 \qquad \times 7$$

$$\boxed{s} = \boxed{n + v}$$

6. 
$$9 = 6 + 3$$

$$\times 5 \qquad \times 5$$

$$\boxed{e} = \boxed{a} + \boxed{r}$$

Write the numerals to complete these number patterns.

7. 
$$\times$$
 3 6 3 + 6 3 9 18 a

12. 
$$\times$$
 50 6 e 9 u v a

Write the numerals to complete these equations.

13. 
$$10 + 5 = \boxed{a}$$

$$\times 3 \times 3$$

$$\boxed{u + c} = \boxed{v}$$

14. 
$$10 + 6 = \boxed{e}$$

$$\times 6$$

$$\boxed{z + n} = \boxed{s}$$

15. 
$$10 + 4 = \boxed{r}$$

$$\times 4 \times 4$$

$$\boxed{s + a} = \boxed{e}$$

17. 
$$20 + 7 = \boxed{a}$$

$$\times 5 \times 5$$

$$\boxed{r + s} = \boxed{e}$$

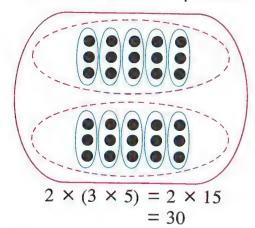
18. 
$$30 + 4 = \boxed{u}$$

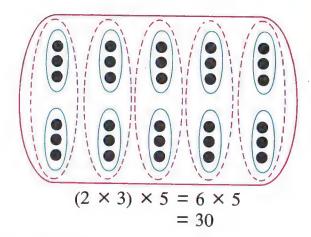
$$\times 6 \times 6 \times 6 = \boxed{x}$$

### Using the grouping rule



study the sets and the equations.





#### EXERCISES

complete the sentence.

1. The grouping rule for multiplication allows us to \_\_? factors any way we wish without changing the product.

olve the equations.

**2.** 
$$2 \times (3 \times 4) = 2 \times |a| = |n|$$

3. 
$$(2 \times 3) \times 4 = \mathbf{c} \times 4 = \mathbf{e}$$

**4.** 
$$2 \times (3 \times 6) = 2 \times e = c$$

**5.** 
$$(2 \times 3) \times 6 = 0 \times 6 = 0$$

**6.** 
$$2 \times (2 \times 5) = 2 \times x = v$$

**7.** 
$$(2 \times 2) \times 5 = v \times 5 = z$$

**8.** 
$$6 \times 7 = (2 \times 3) \times 7 = 2 \times (3 \times 7) = 2 \times \overline{a} = \overline{n}$$

**9.** 
$$6 \times 5 = (2 \times 3) \times 5 = 2 \times (3 \times 5) = 2 \times \boxed{c} = \boxed{0}$$

10. 
$$9 \times 4 = (3 \times 3) \times 4 = 3 \times (3 \times 4) = 3 \times \mathbf{e} = \mathbf{s}$$

11. 
$$4 \times 7 = (2 \times 2) \times 7 = 2 \times (2 \times 7) = 2 \times 0 = r$$

**12.** 
$$8 \times 8 = (4 \times 2) \times 8 = 4 \times (2 \times 8) = 4 \times \mathbf{u} = \mathbf{x}$$

**13.** 
$$3 \times 20 = 3 \times (2 \times 10) = (3 \times 2) \times 10 = \mathbf{a}$$

**14.** 
$$4 \times 30 = 4 \times (3 \times 10) = (4 \times 3) \times 10 = \boxed{e}$$

**15.** 
$$4 \times 60 = 4 \times (6 \times 10) = (4 \times 6) \times 10 = \boxed{c}$$

**16.** 
$$5 \times 70 = 5 \times (7 \times 10) = (5 \times 7) \times 10 = \mathbf{n}$$

## Multiplying hundreds and tens

Study these methods.

$$\begin{vmatrix} 3 \times 210 = (3 \times 200) + (3 \times 10) \\ = 600 + 30 \\ = 630 \end{vmatrix} + \begin{vmatrix} (200 \times 3) = 600 \\ + (10 \times 3) = 30 \\ \hline 210 \times 3 = 630 \end{vmatrix} = \begin{vmatrix} 210 = 200 + 10 \\ \times 3 = \times 3 \\ \hline 600 + 30 = 630 \end{vmatrix}$$

#### EXERCISES

Name the products.

1. 
$$4 \times 220 = 4 \times (200 + 20)$$
  
 $= (4 \times \boxed{s}) + (4 \times \boxed{r})$   
 $= \boxed{a} + \boxed{c}$   
 $= \boxed{o}$   
2.  $3 \times 320 = 3 \times (300 + 20)$   
 $= (3 \times \boxed{x}) + (3 \times \boxed{z})$   
 $= \boxed{e} + \boxed{n}$ 

Write the numerals to complete these equations.

 $\overline{u}$ 

$$\frac{+ (10 \times 4)}{210 \times 4} = \boxed{v}$$

3.  $(200 \times 4) = n$ 

6. 
$$320 = 300 + 20$$
  
 $\times 2$   
 $\boxed{a + \boxed{x}} = \boxed{c}$ 

8. 
$$230 = 200 + 30$$
  
 $\times 3$   
 $\boxed{s} + \boxed{v} = \boxed{z}$ 

4. 
$$(300 \times 3) = \boxed{s}$$
  
 $+ (20 \times 3) = \boxed{u}$   
 $320 \times 3 = \boxed{n}$ 
5.  $(100 \times 4) = \boxed{a}$   
 $+ (20 \times 4) = \boxed{s}$   
 $120 \times 4 = \boxed{v}$ 

7. 
$$430 = 400 + 30$$
  
 $\times 2$   
 $x = 2$   
 $x = 2$   
 $x = 1$ 

9. 
$$240 = 200 + 40$$
  
 $\times 2$   
 $x = 200 + 40$   
 $x = 200 + 40$   
 $x = 200 + 40$   
 $x = 200 + 40$ 

## Multiplying hundreds, tens, and ones



Study the equations and the number pattern.

$$3 \times 232 = 3 \times (200 + 30 + 2)$$

$$= (3 \times 200) + (3 \times 30) + (3 \times 2)$$

$$= 600 + 90 + 6$$

$$= 696$$

$$(200 \times 3) = 600$$

$$(30 \times 3) = 90$$

$$+(2 \times 3) = 6$$

$$232 \times 3 = 696$$

$$232 \times 3 = 696$$

#### EXERCISES

Use equations to name these products.

1. 
$$3 \times 221 = a$$

**2.** 
$$4 \times 112 = c$$

3. 
$$4 \times 221 = [n]$$

Write the numerals to complete these equations.

4. 
$$(300 \times 3) = \mathbf{n}$$
  
 $(10 \times 3) = \mathbf{o}$   
 $+ (3 \times 3) = \mathbf{r}$   
 $313 \times 3 = \mathbf{s}$ 

5. 
$$(400 \times 2) = \overline{v}$$

$$(20 \times 2) = \overline{r}$$

$$+ (3 \times 2) = \overline{s}$$

$$423 \times 2 = \overline{u}$$

6. 
$$(200 \times 3) = \boxed{s}$$
  
 $(30 \times 3) = \boxed{v}$   
 $+ (3 \times 3) = \boxed{x}$   
 $\boxed{233 \times 3} = \boxed{z}$ 

Write the numerals to complete these number patterns.

10-12. Use a number pattern like the one above to name the products in exercises 1-3.

solve these problems.

- **13.** Each row in the baseball park can seat 212 people.
  - a. How many people will 2 rows seat?
  - **b.** How many people will 4 rows seat?
- 14. Suppose there are 111 people sitting in each row. How many people are there in 5 rows?

## Multiplying hundreds, tens, and ones

Study these methods.

Equation method $3 \times 321 = 3 \times (300 + 20 + 1)$ $= (3 \times 300) + (3 \times 20) + (3 \times 1)$ $= 900 + 60 + 3$ $= 963$	Four-step method 321 × 3 60 900 963	Short method $ \begin{array}{r} 321 \\ \times 3 \\ \hline 963 \end{array} $
700	963	

#### EXERCISES

Name each product by the equation method, by the 4-step method, and by the short method.

1. 
$$3 \times 223 = a$$

**2.** 
$$3 \times 211 = C$$

3. 
$$2 \times 243 = n$$

4. 
$$3 \times 333 = [r]$$

5. 
$$4 \times 212 = x$$

**6.** 
$$2 \times 434 = \mathbf{z}$$

Solve the problems.

- 22. How many grapefruit in 4 boxes, if each box contains 112 grapefruit?
- 23. There are 144 hours in 6 days. Use this fact to name the number of hours in 12 days.

258 (two hundred fifty-eight)

## Multiplying tens and ones



tudy these methods.

#### **Equation method**

$$2 \times 36 = 2 \times (30 + 6)$$

$$= (2 \times 30) + (2 \times 6)$$

$$= 60 + 12$$

$$= 72$$

### Three-step method

 $\begin{array}{r}
 36 \\
 \times 2 \\
 \hline
 12 \\
 \hline
 60 \\
 \hline
 72
 \end{array}$ 

#### EXERCISES

lse the equation method to name these products.

1. 
$$2 \times 16 = a$$

**2.** 
$$3 \times 24 = e$$

3. 
$$2 \times 37 = c$$

4. 
$$2 \times 48 = n$$

5. 
$$4 \times 19 = r$$

**6.** 
$$3 \times 18 = s$$

7. 
$$4 \times 17 = u$$

8. 
$$2 \times 49 = v$$

9. 
$$4 \times 15 = 2$$

se the 3-step method to name the products in xercises 1 to 9.

se the 3-step method to name these products.

olve these problems.

- **24.** There are 24 hours in one day. How many hours are there in 4 days?
- **25.** If 6 boxes each contain 16 balls, how many balls are there in all?
- **26.** There are 14 days in a fortnight. How many days are there in 5 fortnights?
- **27.** There are 25 cents in 1 quarter. How many cents are there in 3 quarters?

### Multiplying hundreds, tens, and ones

Study these methods.

### **Equation** method

$$3 \times 324 = 3 \times (300 + 20 + 4)$$
  
=  $(3 \times 300) + (3 \times 20) + (3 \times 4)$   
=  $900 + 60 + 12$   
=  $972$ 

#### Four-step method

972

#### EXERCISES

Use the equation method to name these products.

1. 
$$2 \times 148 = a$$

**2.** 
$$3 \times 225 = n$$

3. 
$$5 \times 117 = S$$

4. 
$$3 \times 227 = [e]$$

5. 
$$2 \times 246 = r$$

**6.** 
$$4 \times 218 = \overline{u}$$

Use the 4-step method to name these products.

Solve these problems.

- **25.** If you had 3 boxes, each containing 125 books, how many books would you have in all?
- **26.** If there are 119 children in grade 3 and each one has 5 books, how many books is that in all?
- **27.** How many crayons would be needed to give 4 crayons to each of 119 children in grade 3?



study these short methods.

#### A. Addition

1 39 **Step 1:** 9 + 8 = 17+ 48

Write 7 in the ones' place, 1 in the tens' place.

87 **Step 2:** 1 + 3 + 4 = 8

Write 8 in the tens' place.

#### В. Multiplication with tens and ones

1 36 **Step 1:**  $2 \times 6 = 12$ 

Write 2 in the ones' place, 1 in the tens' place.

 $\times 2$ 

**Step 2:**  $(2 \times 3) + 1 = 7$ 

Write 7 in the tens' place.

#### C. Multiplication with hundreds, tens and ones

436

**Step 1:**  $2 \times 6 = 12$ 

 $\times 2$ 

872

**Step 3:**  $2 \times 4 = 8$ 

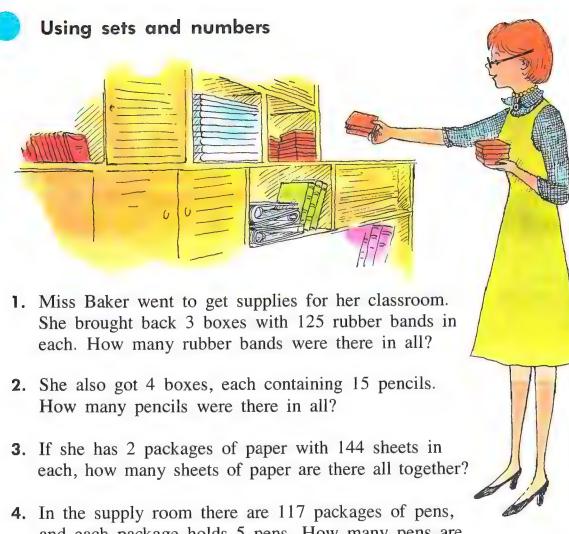
Write 2 in the ones' place, 1 in the tens' place.

**Step 2:**  $2 \times 3 = 6, 6 + 1 = 7$  Write 7 in the tens' place. Write 8 in the hundreds' place.

#### EXERCISES

Ise the short method to name these products.

 $\times 4$ 



- **4.** In the supply room there are 117 packages of pens, and each package holds 5 pens. How many pens are there in all?
- **5.** Each of the shelves in the supply room holds 32 boxes of paper. How many boxes of paper are there on the 6 shelves?
- **6.** Miss Baker brought back 6 packages of construction paper, and each package contains 72 sheets of paper. How many pieces of paper are there in all?
- 7. Crayons come in packages of 8. If there are 106 packages in the supply room, how many crayons is that all together?

### Two stage problems

ms

Bill has 3 boxes; each box contains 4 toy cowboys. He has two other boxes which contain 5 toy horses each. How many toys does Bill have all together?

To solve this problem, use the 5-step method.

Step 1: What are the sets in the problem? (There are 3 sets of 4 and 2 sets of 5.)

Step 2: What is happening to the sets? (They are being combined.)

Step 3: Write the equation:  $(3 \times 4) + (2 \times 5) = \overline{a}$ 

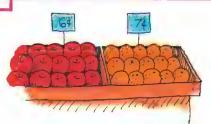
**Step 4:** Solve the equation:  $(3 \times 4) + (2 \times 5) = 12 + 10 = 22$ 

Step 5: Use the solution to answer the question. (Bill has 22 toys.)

#### EXERCISES

Solve these problems.

- 1. Bob buys 4 apples at 6 cents each and 4 oranges at 7 cents each. How much does he spend in all?
- 2. If 6 cars had 5 people each, and 2 cars had 4 people each, how many people are there in all?
- **3.** Two buses carried 30 children each, and three buses carried 40 children each. How many children were carried in all?
- 4. Mario's Boy Scout troop hiked 14 miles a day for 4 days. Larry's troop hiked 17 miles a day for 3 days. How much farther did Mario's troop hike?
- 5. On Monday Lee sold 37 newspapers, on Tuesday 43, on Wednesday 46, and on Thursday 29. How many more did he sell on Monday and Tuesday than on Wednesday and Thursday?







# Multiplication

Study these methods.

#### Three-step method

#### Short method

5 **Step 1:**  $8 \times 7 = 56$ . Write 6  $\times 8$ in the ones' place, 5 376 in the tens' place. Step 2:  $8 \times 4 = 32$ 32 + 5 = 37Write 7 in the tens' place,

3 in the hundreds' place.

68

 $\times$  7

7.

 $\times$  7

#### EXERCISES

Name these products using the 3-step method.

To practice step 2 in the short method, we have written an equation above each example. First solve the equation, then use the sum to complete the product using the short method.

**9.** 
$$32 + 3 = \boxed{a}$$
**9.**  $32 + 6 = \boxed{c}$ 
**10.**  $32 + 7 = \boxed{s}$ 

$$44 \times 8 \times 8 \times 8$$

$$\times 8 \times 8$$

11. 
$$30 + 5 = \boxed{s}$$

$$59 \times 6 \times 6$$

$$24 + 4 = \boxed{u}$$

$$47 \times 6 \times 7$$

$$25 \times 6 \times 7$$

$$35 \times 7$$

## Practice page



Name the sums.

Name the products using the short method.

Name the products using the short method.

### The hobby show



Miss Lee's class had a hobby show. The children brought to school some of the things they collected.

- 1. Jonny has a coin collection. Each book holds 36 coins, and he has 3 full books. How many coins has Jonny in all?
- 2. Sandi collects picture postcards. She has 7 boxes, and each box contains 35 cards. How many cards does she have in all?
- 3. Eric's stamp book has 8 pages with 45 stamps on each page. How many stamps are there in the book?
- **4.** Bob collects stamps, also. His stamp book has 9 pages with 42 stamps on each page. Who has more stamps, Bob or Eric? How many more?
- **5.** Hope has a 124 page scrap-book of animal pictures. If there are 4 pictures on each page, how many pictures are there in all?

266 (two hundred sixty-six)

#### Practice page



Write four equations for each of these sets of two addends and a sum.

Write four equations for each of these sets of two factors and a product.

Complete these equations. Write +, -,  $\times$  or  $\div$  to replace each  $\triangle$ .

17. 
$$7 \triangle 7 = 49$$
 18.  $8 \triangle 5 = 3$  19.  $8 \triangle 5 = 13$ 

**18.** 
$$8 \triangle 5 = 3$$

**19.** 
$$8 \triangle 5 = 13$$

**20.** 
$$32 \triangle 8 = 4$$
 **21.**  $7 \triangle 3 = 21$  **22.**  $6 \triangle 4 = 24$ 

21. 
$$7 \triangle 3 = 21$$

**22.** 
$$6 \triangle 4 = 24$$

**23.** 
$$27 \triangle 9 = 3$$

**23.** 
$$27 \overline{\triangle} 9 = 3$$
 **24.**  $27 \triangle 9 = 18$  **25.**  $3 \overline{\triangle} 3 = 1$ 

**25.** 
$$3 \triangle 3 = 1$$

Complete these equations. Write  $\leq$ , > or = to replace each  $\bigcirc$ .

**26.** 
$$3 + 4 \bigcirc 3 \times 4$$

**27.** 
$$2 + 2 \bigcirc 2 \times 2$$

**28.** 
$$12 - 6 \bigcirc 12 \div 2$$

**29.** 
$$7 \times 2 \bigcirc 6 + 8$$

**30.** 
$$4 + 10 \bigcirc 6 \times 2$$

**31.** 
$$2 \times 6$$
 ( )  $3 \times 4$ 

**32.** 
$$4 \times 10 \bigcirc 5 \times 8$$

**31.** 
$$2 \times 6 \bigcirc 3 \times 4$$
  
**33.**  $6 + 7 \bigcirc 6 \times 2$ 

Just For Fun

#### **An Addition Race**

Gay and Anna used to have addition races. On their work sheets they would show one number and then race one another adding another number until their sum reached 100. One day, for instance, they both wrote 7 on their papers and kept adding 6 until the sum passed 100. That day Gay won. On her paper she had written: 7, 13, 19, 25, 31, 37, 43, 49, 55, 61, 67, 73, 79, 85, 91, 97, 103.

Can you discover a good method of checking this addition?

Can you invent a subtraction race?

#### Doing and undoing



Lynn filled the can. Now she is emptying it.





Joining a set of 5 with a set of 3 forms a set of 8.





Separating a set of 5 from a set of 8 leaves a set of 3.

#### EXERCISES

Solve the equations.

1. 
$$4 + 3 = 7$$
, so  $7 - 3 = a$ 

3. 
$$4 + 8 = 12$$
, so  $12 - 8 = \overline{c}$ 

5. 
$$36 + 12 = 48$$
, so  $48 - 12 = \boxed{e}$ 

**7.** 
$$436 + 584 = 1020$$
, so  $1020 - 584 = \boxed{v}$ 

**2.** 
$$4 + 6 = 10$$
, so  $10 - 6 = \boxed{r}$ 

**3.** 
$$4 + 8 = 12$$
, so  $12 - 8 = \boxed{c}$  **4.**  $16 + 9 = 25$ , so  $25 - 9 = \boxed{x}$ 

**6.** 
$$48 + 12 = 60$$
, so  $60 - 12 = \overline{u}$ 

**8.** 
$$986 + 394 = 1380$$
, so  $1380 - 394 = \boxed{z}$ 

Use the number line to help you answer the questions.

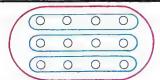


- 9. Start at 7 on the number line; move 3 to the right, then 3 to the left. You stop at  $\overline{a}$ .
- 10. Start at 9 on the number line; move 5 to the left, then 5 to the right. You stop at r.

11.	start at:	2	2	7	6	9	10	11	9
	move to the right:	4	8	5	6	4	2	2	5
	stop at:	a	r	C	S	e	u	υ	n
	move to the left:	4	8	5	6	4	2	2	5
	stop at:	x	z	a	r	v	n	c	S

### Doing and undoing





John put 3 sets of 4 together to form a set of 12.

0	0	0	0
0	0	0	0
0	0	0	0

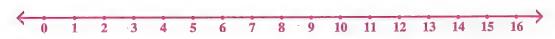
Then John separated the set of 12 to form 3 sets of 4.

#### EXERCISES

Write the numeral to go in each  $\square$ .

- 1. 3 sets of 5 form 1 set of  $\overline{a}$ .
- 3. 6 sets of 4 form 1 set of  $\overline{c}$ .
- **2.** 4 sets of 7 form 1 set of  $\overline{n}$ .
- **4.** 1 set of 15 forms 3 sets of [n].
- 5. 2 sets of 9 form 1 set of [e]. 6. 1 set of 24 forms [r] sets of 4.

Jse the number line to solve the equations.



7.  $4 \times |a| = 16$ 

- 8.  $16 \div 4 = |x|$
- 9. If you start at zero on the number line, make 6 moves of 5 to the right, then 6 moves of 5 to the left, you stop at v.

Jse the number line to help you complete the chart.



Start at zero.

10. Move to the right

number of moves:	4	5	2	3	6	7	9	8
size of each move:	4	3	4	3	3	2	2	2
you stop at:	16	a	C	e	n	x	r	S
number of moves:	4	5	2	3	6	7	9	8
size of each move:	4	3	4	3	3	2	2	2
you stop at:	0	u	v	$\boldsymbol{x}$	z	a	C	n

Move to the left

11. Can you state the general rule?

Division undoes ?...

#### Division "undoes" multiplication

$$5 \times 6 = 30$$
  $30 \div 6 = 5$   
 $6 \times 5 = 30$   $30 \div 5 = 6$ 

These four equations form a Family of Facts.

#### EXERCISES

Complete these equations.

1. 
$$4 \times 3 = 12$$
, so  $12 \div 3 = \overline{a}$  2.  $5 \times 3 = 15$ , so  $15 \div 3 = \overline{u}$ 

**2.** 
$$5 \times 3 = 15$$
, so  $15 \div 3 = [$ 

**3.** 
$$7 \times 5 = 35$$
, so  $35 \div 5 = \boxed{c}$ 

**4.** 
$$6 \times 9 = 54$$
, so  $54 \div 6 = \boxed{v}$ 

**5.** 
$$6 \times 24 = 144$$
, so  $144 \div 6 = \boxed{e}$ 

**6.** 
$$18 \times 3 = 54$$
, so  $54 \div 3 = \boxed{z}$ 

Write the numerals to complete these patterns.

8.	product	12	30	14	24	45	18	16	27
	factor	4	6	2	8	9	6	2	3
	factor	3	a	u	c	r	e	z	n

9. What do you notice about the first row in question 7 and the last row in question 8?

Write two equations to complete the family for each of these pairs of equations.

**10.** 
$$3 \times 6 = 18, 6 \times 3 = 18$$

11. 
$$28 \div 4 = 7, 28 \div 7 = 4$$

12. 
$$3 \times 9 = 27, 9 \times 3 = 27$$

13. 
$$32 \div 8 = 4, 32 \div 4 = 8$$

**14.** 
$$3 \times 7 = 21$$
,  $7 \times 3 = 21$ 

**15.** 
$$8 \div 4 = 2$$
,  $8 \div 2 = 4$ 

**16.** 
$$2 \times 5 = 10, 5 \times 2 = 10$$

17. 
$$16 \div 8 = 2, 16 \div 2 = 8$$

Complete the sentence.

**18.** Division is the opposite of \_\_?

270 (two hundred seventy)

## Different ways of multiplying and dividing



Multiplication is naming the product.

$$3 \times 4 = 12$$
 $12 \div 3 = 4$ 
 $\times \mid 4$ 

Division is naming a factor.

$$3 \times 4 = 12$$
 $12 \div 3 = 4$ 

$$\times | 4 \\
\hline
3 | 12$$
 $3)12$ 

#### EXERCISES

lame the products or missing factors.

1. 
$$3 \times 8 = a$$

**2.** 
$$6 \times 7 = 0$$

3. 
$$7 \times 8 = c$$

4. 
$$e \div 3 = 6$$

5. 
$$n \div 2 = 8$$

**6.** 
$$s \div 3 = 7$$

7. 
$$9 \times 7 = r$$

8. 
$$u \div 4 = 5$$

9. 
$$36 \div z = 6$$

10. 
$$12 \div s = 6$$

11. 
$$36 \div 4 = v$$

12. 
$$32 \div [c] = 4$$

13. 
$$[e] \times 6 = 36$$
 14.  $3 \times [n] = 27$ 

14. 
$$3 \times n = 27$$

15. 
$$o \times 8 = 72$$

complete the number charts, naming the products.

omplete the number charts, naming the missing factors.

19.	×	z	a	C	
	4	16	24	32	

20.	×	$\boldsymbol{n}$	e	u		
	8	40	16	48		

lame the products.

28. 
$$4)$$
  $29.$   $7)$   $\overline{v}$  30.  $9)$   $\overline{x}$  31.  $5)$   $\overline{z}$  32.  $6)$   $\overline{r}$  33.  $8)$   $\overline{a}$ 

**32.** 6) 
$$r$$

33. 
$$8) | \overline{a} |$$

lame the missing factors.

**34.** 
$$2)\overline{12}$$
 **35.**  $3)\overline{15}$  **36.**  $4)\overline{20}$  **37.**  $9)\overline{18}$  **38.**  $7)\overline{21}$  **39.**  $6)\overline{36}$ 

**40.** 
$$3)\overline{24}$$
 **41.**  $4)\overline{32}$  **42.**  $5)\overline{40}$  **43.**  $6)\overline{48}$  **44.**  $9)\overline{72}$  **45.**  $3)\overline{24}$ 

#### Subtraction and division

Study these equations.

**a.** 
$$18 - 3 = 15$$
 **b.**  $15 - 3 = 12$ 

c. 
$$12 - 3 = 9$$
 d.  $9 - 3 = 6$ 

e. 
$$6-3=3$$
 f.  $3-3=0$ 

These 6 equations may be combined as follows:

$$18 - (3 + 3 + 3 + 3 + 3 + 3) = 0$$

The 3 was subtracted 6 times.

$$18 - (6 \times 3) = 0$$

In a set of 18 there are 6 sets of 3.

$$18 \div 3 = 6$$

#### EXERCISES

Write the numeral to go in each  $\square$ .

1. 
$$12 - (6 + 6) = \overline{a}$$

1. 
$$12 - (6 + 6) = \overline{a}$$
,  $12 - (\overline{u} \times 6) = 0$ ,  $12 \div 6 = \overline{c}$ ,  $6\overline{)}$ 

**2.** 
$$15 - (5 + 5 + 5) = \overline{e}$$
,  $15 - (\overline{v} \times 5) = 0$ ,  $15 \div 5 = \overline{z}$ ,  $5)\overline{15}$ 

3. 
$$21 - (7 + 7 + 7) = \mathbf{r}$$
,  $21 - (\mathbf{x} \times 7) = 0$ ,  $21 \div 7 = \mathbf{a}$ ,  $7)21$ 

**4.** 
$$12 - (4 + 4 + 4) = \boxed{s}$$
,  $12 - (\boxed{n} \times 4) = 0$ ,  $12 \div 4 = \boxed{r}$ ,  $4)12$ 

Use the equations on the left to complete the divisions on the right.

**5.** 
$$24 = 6 + 6 + 6 + 6$$
 6)24

**6.** 
$$30 = 5 + 5 + 5 + 5 + 5 + 5$$
  $5)30$ 

**7.** 
$$20 = 4 + 4 + 4 + 4 + 4 + 4$$
 4)20

Solve the equations.

**8.** 
$$24 = 6 \times 4$$
, so  $24 \div 4 = \overline{a}$  **9.**  $36 = 4 \times 9$ , so  $36 \div 9 = \overline{n}$ 

**10.** 
$$35 = 7 \times 5$$
, so  $35 \div 5 = \boxed{c}$  **11.**  $42 = 7 \times 6$ , so  $42 \div 6 = \boxed{u}$ 

Name the missing factors.

**12.** 
$$28 \div 4 = \boxed{r}$$
 **13.**  $36 \div 6 = \boxed{c}$  **14.**  $45 \div 5 = \boxed{s}$ 

**15.** 
$$6\overline{\smash{\big)}\,48}$$
 **16.**  $8\overline{\smash{\big)}\,64}$  **17.**  $3\overline{\smash{\big)}\,15}$  **18.**  $9\overline{\smash{\big)}\,45}$  **19.**  $5\overline{\smash{\big)}\,10}$  **20.**  $3\overline{\smash{\big)}\,36}$ 

**21.** 
$$7\overline{)56}$$
 **22.**  $3\overline{)9}$  **23.**  $6\overline{)6}$  **24.**  $7\overline{)42}$  **25.**  $8\overline{)16}$  **26.**  $9\overline{)9}$ 

272 (two hundred seventy-two)

#### Division on the number line





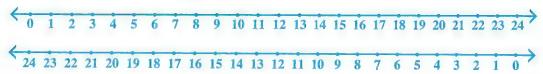
To solve the equation  $24 \div 3 = \square$ , we may count back on the number line. There are 8 moves of 3 from 24 to 0. There are 8 sets of 3 in a set of 24.  $24 \div 3 = 8.$ 

#### EXERCISES

Count back on the number line to find how many:

- 1. sets of 6 in 24
- **2.** sets of 2 in 18 **3.** sets of 7 in 21
- 4. sets of 3 in 12 5. sets of 4 in 20 6. sets of 3 in 21

f you have trouble counting back on your number line ou may use a second number line.



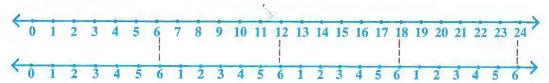
**7.** How many 3's in 24?

**8.** How many 4's in 24?

**9.** How many 8's in 24?

**10.** How many 6's in 24?

ou may cut a number line into segments and use the egments like this:



- 11. How many 6's in 24? 12. How many 6's in 18?

lame the missing factors.

- 13.  $24 \div 4 = |a|$
- **14.**  $20 \div 4 = \boxed{x}$  **15.**  $36 \div 9 = \boxed{c}$
- **16.**  $56 \div 8 = \overline{v}$  **17.**  $63 \div 9 = \overline{n}$
- 18.  $28 \div 4 = [e]$
- **19.**  $2)\overline{14}$  **20.**  $2)\overline{16}$  **21.**  $7)\overline{63}$  **22.**  $7)\overline{49}$  **23.**  $5)\overline{40}$  **24.**  $7)\overline{42}$

- **25.**  $5)\overline{10}$  **26.**  $9)\overline{27}$  **27.**  $3)\overline{24}$  **28.**  $8)\overline{64}$  **29.**  $3)\overline{21}$  **30.**  $9)\overline{81}$

#### Division with remainders



Larry wanted to know how many sets of 6 he could form from a set of 42.

or 
$$\frac{42 \div 6}{\boxed{n}} \times 6 = \frac{\boxed{n}}{42}$$
$$\boxed{n} = 7$$

He could form 7 sets of 6 with 0 remaining.



Gay wanted to know how many sets of 6 she could form from a set of 44.

She wrote  $m \times 6 = 44$ . But she knew that  $7 \times 6 = 42$  and  $8 \times 6 = 48$ . She knew she could not solve her equation with a counting number.

44 = 42 + 2,  $44 = (7 \times 6) + 2$ . She could form 7 sets of 6 with 2 remaining.

#### EXERCISES

#### Name the number that is

- 1. 2 greater than  $(3 \times 6)$ .
- **3.** 3 greater than  $(5 \times 7)$ .
- **5.** 4 greater than  $(5 \times 6)$ .
- 7. 5 greater than  $(6 \times 6)$ .
- **9.** 6 greater than  $(7 \times 7)$ .
- 11. 7 greater than  $(8 \times 8)$ .

- 2. 2 greater than  $(4 \times 7)$ .
- **4.** 3 greater than  $(6 \times 4)$ .
- **6.** 4 greater than  $(7 \times 8)$ .
- **8.** 5 greater than  $(6 \times 8)$ .
- 10. 6 greater than  $(8 \times 9)$ .
- 12. 7 greater than  $(9 \times 9)$ .

#### Solve the equations.

13. 
$$14 = (3 \times 4) + \boxed{a}$$

**14.** 
$$18 = (4 \times 4) + \mathbf{r}$$

**15.** 
$$19 = (6 \times 3) + \overline{c}$$

**16.** 
$$16 = (3 \times 5) + \mathbf{e}$$

17. 
$$20 = (3 \times 6) + \overline{u}$$

18. 
$$17 = (4 \times 4) + \overline{v}$$

19. 
$$23 = (4 \times 5) + x$$

**20.** 
$$26 = (4 \times 6) + \overline{a}$$

**21.** 
$$11 = (2 \times 5) + \boxed{c}$$

**22.** 
$$31 = (4 \times 7) + \mathbf{r}$$

**23.** 
$$33 = (5 \times 6) + n$$

**24.** 
$$13 = (3 \times 4) + \overline{r}$$

**25.** 
$$43 = (8 \times 5) + \overline{a}$$

**26.** 
$$42 = (8 \times 5) + \mathbf{r}$$

**27.** 
$$29 = (7 \times 4) + \boxed{s}$$

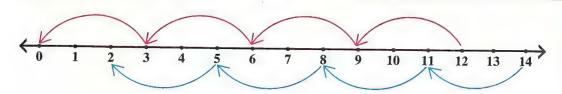
**28.** 
$$66 = (8 \times 8) + \mathbf{u}$$

**29.** 
$$37 = (7 \times 5) + \overline{v}$$

**27.** 
$$29 = (7 \times 4) + 8$$
  
**30.**  $38 = (7 \times 5) + 6$ 

#### Division on the number line





 $12 \div 3 = n$  can be solved with a counting number.

Count back by threes from 12.

$$12 - (4 \times 3) = 0$$

$$12 \div 3 = 4$$

There are 4 sets of 3 in a set of 12 with 0 remaining.

$$12 = 4 \times 3$$

 $14 \div 3 = \mathbf{n}$  cannot be solved with a counting number.

Count back by threes from 14.

$$14 - (4 \times 3) = 14 - 12$$
  
= 2

There are 4 sets of 3 in a set of 14 with 2 remaining.  $14 = (4 \times 3) + 2$ 

### EXERCISES

olve the equations.

1. 
$$11 - (4 \times 2) = [\overline{a}]$$

2. 
$$9 - (4 \times 2) = \overline{c}$$

**2.** 
$$9 - (4 \times 2) = \boxed{c}$$
 **3.**  $13 - (5 \times 2) = \boxed{e}$ 

**4.** 
$$12 - (5 \times 2) = \mathbf{r}$$

5. 
$$10 - (3 \times 3) = \mathbf{r}$$
 6.

**6.** 
$$7 - (2 \times 3) = s$$

7. 
$$13 - (4 \times 3) = \overline{v}$$

8. 
$$14 - (2 \times 6) = |\overline{x}|$$

9. 
$$18 - (5 \times 3) = [z]$$

**10.** 
$$16 - (3 \times 5) = C$$

11. 
$$15 - (6 \times 2) = e$$

12. 
$$21 - (6 \times 3) = n$$

13: 
$$17 - (7 \times 2) = \mathbf{r}$$

14. 
$$29 - (4 \times 7) = s$$

15. 
$$33 - (6 \times 5) = \overline{u}$$

**16.** 
$$^{\hat{}}$$
 19 - (8 × 2) =  $\bar{x}$ 

17. 
$$29 - (4 \times 6) = \mathbf{z}$$

18. 
$$37 - (4 \times 9) = a$$

19. 
$$25 - (4 \times 6) = \mathbf{Z}$$

**20.** 
$$46 - (9 \times 5) = \overline{v}$$

**21.** 
$$64 - (7 \times 9) = \boxed{c}$$

**22.** 
$$13 - (a \times 4) = 1$$

**23.** 
$$16 - (C \times 3) = 1$$

**24.** 
$$13 - (\mathbf{z} \times 5) = 3$$

**25.** 
$$10 - (\mathbf{n} \times 3) = 1$$

**26.** 
$$12 - (x \times 5) = 2$$

**27.** 
$$11 - (r \times 4) = 3$$

**28.** 
$$9 - (S \times 4) = 1$$

**29.** 
$$10 - (\mathbf{u} \times 4) = 2$$

**30.** 
$$16 - (\overline{v} \times 6) = 4$$

31. 
$$15 - (x \times 4) = 3$$

**32.** 
$$16 - (\mathbf{z} \times 5) = 1$$

**33.** 
$$20 - (\mathbf{a} \times 8) = 4$$

**34.** 
$$16 = (2 \times 7) + \overline{a}$$

**35.** 
$$14 = (2 \times 5) + \boxed{c}$$

**36.** 
$$15 = (2 \times 6) + \boxed{e}$$

37. 
$$8 = (2 \times 3) + \mathbf{n}$$

**38.** 
$$11 = (2 \times 4) + \bar{x}$$

**39.** 
$$7 = (2 \times 3) + \mathbf{r}$$

**10.** 
$$13 = (\mathbf{S} \times 6) + 1$$

**41.** 
$$14 = (\underline{u} \times 6) + 2$$

**42.** 
$$15 = (\overline{v} \times 7) + 1$$

13. 
$$30 = (x \times 7) + 2$$

**44.** 
$$27 = (\mathbf{z} \times 5) + 2$$

**45.** 
$$35 = (\overline{a} \times 8) + 3$$

#### Division with remainders

Study these ways of showing that there are 7 sets of 6 in a set of 44 with 2 left over.

$44 = (7 \times 6) + 2$	7)44
$44 - (7 \times 6) = 2$	$\frac{42}{2}$

## EXERCISES

Write the numeral to replace each  $\square$ .

4. 
$$4)\overline{23}$$
 $20$ 
 $\overline{n}$ 

8. 
$$5)\overline{27}$$
 $25$ 
 $\overline{u}$ 

18. 
$$4)\overline{27}$$
 $\overline{v}$ 
3

19. 9)48 
$$\frac{x}{3}$$

20. 
$$8)50$$
  $\boxed{z}$   $\boxed{2}$ 

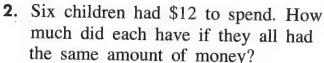
22. 
$$7)\overline{18}$$
 $\frac{14}{4}$ 

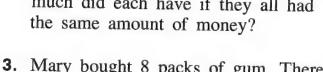
23. 
$$4)22$$
 $\frac{20}{2}$ 

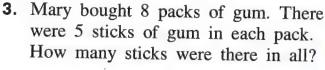
25. 
$$9)31$$
 $\frac{27}{4}$ 

## Using sets and numbers

1. Jan had 46 crayons which she wanted to put into sets of 6. How many complete sets could she make?

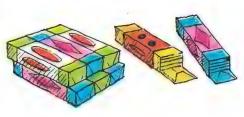




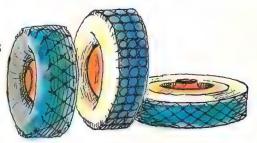


- **4.** There are 7 days in one week. How many complete weeks are there in 35 days? in 37 days?
- 5. We have 35 sheets of paper for 8 children. How many sheets are there for each child? How many sheets are there left over?
- **6.** Each pupil needs 8 sheets of paper. How many sheets are needed for 9 pupils?
- 7. Thirty-three tires make how many sets of 4? How many tires are left over?
- **8.** Six children each got 5 apples, and there were 3 apples left over. How many apples were there in all?
- **9.** If each child needs 8 crayons, how many crayons are needed for 8 children?











### Numerals with money

\$3.97 means "three dollars and ninety-seven cents, three hundred ninety-seven cents, or 397¢."

The point is used to separate the dollars from the cents.



#### EXERCISES

How many cents in

- 1. one dollar
- 3. three dollars and ninety-seven cents
- 4. \$2.00
- **5**. \$4.00
- **6.** \$5.00
- **7**. \$3.00

2. three dollars

8, \$7.00

- 9. \$2.25
- **10.** \$3.20
- 11. \$4.75
- **12.** \$6.54
- **13**. \$5.39

Write these amounts of money using the \$ sign and the point as at the top of the page.

- 14. three dollars and twenty-five cents
- 16. one dollar and forty-eight cents
- 18. six hundred thirty-nine cents

- 15. four dollars and fifty cents
- 17. ninety-eight cents
- 19. four dollars and ten cents

In each of these pairs name the greater amount.

**20**. 320¢, \$3.30

**21.** \$6.45, 650¢

**22.** 250¢, \$2.00

23. \$2.65, 725¢

Which of these is less than a dollar?

- **24.** \$2.64 **25.** \$.74 **26.** \$1.10
- **27.** \$.90
- **28.** \$.40

Complete this chart.

Number of Coins

Value in Cents

Short Form

Dollars	Dimes	Cents	Hundreds	Tens	Ones	Using \$ and .
4	2	3	4	2	3	\$4.23
5	7	6	5	а	r	С
5	0	e	5	2	6	S
n	u	v	z	x	e	\$3.29

## Working with money



Study these examples.

To name a sum.

В To name a missing addend.

To name a product.

\$7.45

$$$4.46$$
 $-1.25$ 
 $\hline$ 
 $$3.21$ 

#### EXERCISES

Name the total amounts.

$$\begin{array}{c}
423 \text{ cents} \\
+ 235 \text{ cents}
\end{array}$$

2.41

|a|

+3.25

+2.24

e

$$+1.72$$

lame the missing addends.

|c|

$$-2.32$$

$$\frac{232}{n}$$
 cents

$$\frac{-2.32}{\$ [0]}$$

$$-2.68$$

$$\frac{-.20}{\$}$$

$$\frac{-2.68}{\$}$$

ame the total amounts.

$$434 \, \stackrel{\times}{\times} 2$$

\$ 0

14. 
$$$1.15$$
  
 $\times 6$   
 $\boxed{\$}$ 

$$\frac{\times 4}{\$}$$

$$\frac{\times 3}{\$}$$

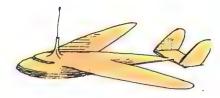
$$\frac{\times 3}{\$}$$

$$\frac{\times 3}{\$}$$

## Figuring the cost

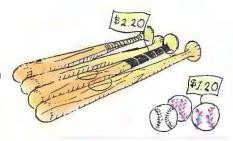
- 1. A coat costs \$17.40, a hat costs \$6.75. How much do they cost together?
- 2. In problem 1 how much more did the coat cost than the hat?
- **3.** One model airplane costs \$3.20. What would 6 model airplanes cost?
- 4. If you spent \$17.46 at the clothing store and \$6.63 at the hobby shop, how much did you spend in all?
- 5. A red bicycle costs \$25.76, and a green one costs \$23.42. How much more does the red bicycle cost?
- 6. If you spent \$3.24, \$4.28, and \$10.31, how much did you spend in all?
- 7. Would 4 ties at \$1.20 each cost more than 3 ties at \$1.50 each?
- 8. What would be the total cost of 4 bats at \$2.20 each and 3 balls at \$1.20 each?
- 9. What would be the total cost of 2 pairs of tennis shoes at \$4.00 each, and 3 pairs of leather shoes at \$6.80 each?











### LOOKING WAY BACK



Write 5-digit numerals for

- 1. sixteen thousand, two hundred twenty-three
- 2. forty thousand, six hundred seventy
- 3. thirty thousand, six hundred thirty-nine
- 4. eighty-one thousand, five hundred

Name the sums.

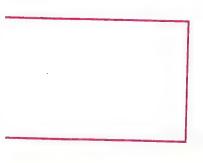
Name the missing addends.

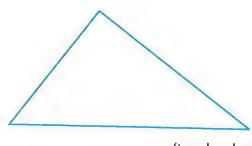
n the numeral 46,954, name

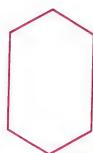
- 23. the tens' digit 24. the thousands' digit 25. the ones' digit

Answer the questions.

- 26. How many sides in all for 3 rectangles, 2 squares, 3 triangles?
- 27. What is the total length of the sides of each of these figures?







### CHECKING UP

Complete these number patterns.

1.	×	6	8	14
	4	a	r	C

3. 
$$\times$$
 20 8 28 4  $v$   $e$   $n$ 

Name these products.

**4.** 
$$4 \times 41$$

8. 
$$4 \times 17$$

10. 
$$8 \times 23$$

Name these missing factors.

Write the numerals to complete these examples.

Solve these equations.

**22.** 
$$13 = (3 \times 4) + \boxed{a}$$

**23.** 
$$23 = (5 \times 4) +$$

**22.** 
$$13 = (3 \times 4) + \boxed{a}$$
 **23.**  $23 = (5 \times 4) + \boxed{s}$  **24.**  $70 = (8 \times 8) + \boxed{c}$ 

**25.** 
$$22 = (5 \times 4) + \mathbf{n}$$

**26.** 
$$46 = (7 \times 6) + \underline{e}$$

**25.** 
$$22 = (5 \times 4) + \mathbf{n}$$
 **26.**  $46 = (7 \times 6) + \mathbf{e}$  **27.**  $51 = (7 \times 7) + \mathbf{o}$ 

Name these products.

Name these amounts.

 $\times$  5

$$\times 4$$

### LOOKING BACK



The Common Factor Rule allows us to add, then multiply, when the addends have a common factor

$$(5 \times 4) + (3 \times 4) = (5 + 3) \times 4 = 8 \times 4 = 32$$
, or multiply, then add.

$$(5 \times 4) + (3 \times 4) = 20 + 12 = 32$$

Ise two methods to rename each of these sums.

1. 
$$(3 \times 9) + (2 \times 9)$$

**2.** 
$$(4 \times 6) + (5 \times 6)$$

**3.** 
$$(7 \times 3) + (7 \times 5)$$

**4.** 
$$(8 \times 4) + (8 \times 2)$$

Vrite the numerals to complete these number patterns.

5. 
$$5 + 6 = 11$$
  
 $\times 5 \times 5$   
 $\boxed{n + 0 = r}$ 

7. 
$$\times$$
 8 9 17 6  $x$   $z$   $a$ 

The short method for multiplying with one factor greater than ten is shown in these two examples.

Without renaming ones as tens

With renaming ones as tens

$$\frac{23}{\times 3} \times 3 = 9 \text{ (ones)}$$
  
 $\frac{69}{3} \times 2 = 6 \text{ (tens)}$ 

$$23$$
 $\times 4$ 
 $4 \times 3 = 12 \text{ (1 ten, 2 ones)}$ 
 $92$ 
 $(4 \times 2) + 1 = 9 \text{ (tens)}$ 

lame these products.

Division is the opposite of multiplication; for example,  $3 \times 7 = 21$  so  $21 \div 7 = 3$ .

ame these missing factors.

14. 
$$3 \times a = 27$$

15. 
$$r \times 4 = 20$$

16. 
$$5 \times [c] = 35$$

omplete the equations.

17. 
$$17 = (4 \times 4) + |a|$$

17. 
$$17 = (4 \times 4) + \boxed{a}$$
 18.  $21 = (5 \times 4) + \boxed{e}$  19.  $31 = (6 \times 5) + \boxed{c}$ 

19. 
$$31 = (6 \times 5) + \overline{c}$$

## CHAPTER 10 · Geometry, Graphs



#### Circles



 $\boldsymbol{A}$ 



R

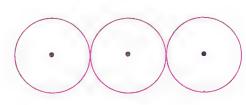


C

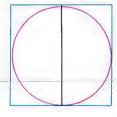
- 1. Are all these closed curves? How do you tell?
- **2.** What is the special name for curve *C*? What can you say about the shape of curves like curve *C*?



**3.** What can you say about the centers of these circles?



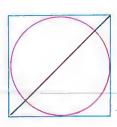
**4.** What can you say about the radius of each of these circles?



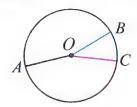
**5.** What can you say about the diameter of this circle and the sides of the square?



**7.** What can you say about the spokes of this wheel?



**6.** What can you say about the diameter of this circle and the diagonal of the square?



**8.** What can you say about  $\overline{OA}$ ,  $\overline{OB}$ ,  $\overline{OC}$ ?

284 (two hundred eighty-four)



- 1. Draw a circle with radius
  - **a.**  $1\frac{1}{2}$  inches

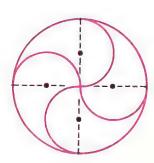
- **b.** 2 inches
- c.  $2\frac{1}{2}$  inches

(If you have forgotten how to draw a circle, look back to page 84.)

- **2.** Draw a line segment AB 2 inches long.
  - a. Draw a circle with A as the center, and B as a point on the circle.
  - **b.** Draw a circle which has B as the center and A as a point on the circle.
  - **c.** What is the radius of each circle you have drawn?
  - d. What is the diameter of each circle?
- **3.** The circles you have drawn for exercise 2 meet in two points.
  - $\alpha$ . Label these points C and D.
  - **b.** Draw  $\overline{AC}$ ,  $\overline{CB}$ ,  $\overline{BD}$ ,  $\overline{DA}$ .
  - **c.** What is the name of the figure you have just drawn?
- **4.** Draw a circle with radius 2 inches. Then mark off segments around the circle with the radius.

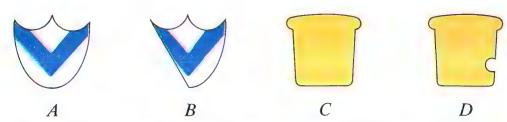
Did you mark off the radius 6 times? Draw line segments to join the points on the circle. How many sides are there in the new figure?

**5.** Here is a pattern we have made with circles. Can you draw one like it?



### Matching sides

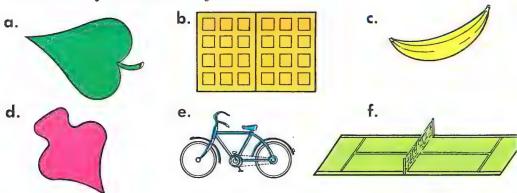
- 1. Draw a circle with a radius of 2 inches. Cut out the circular region and draw a diameter. Fold your paper along the diameter. Do the halves of the circle match? Does one half of the circle fit on top of the other?
- 2. Draw a square. Cut out the square region and draw a diagonal. Fold your paper along the diagonal. Do the halves of the square match?
- 3. Look at these shapes.

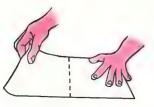


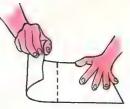
What do you notice about a. the two shields on the left? b. the two pieces of toast on the right?

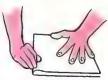
The parts of A and C look as if they match. Shapes whose parts match are called **symmetrical**. Some objects are symmetrical, others are not. Name some symmetrical objects in your classroom.

4. Name the symmetrical objects.









Bob thought he had a rule to decide whether a shape is symmetrical. He said, "You pick out a line which you think will separate the shape into two matching halves. Then fold the paper along the line. If the two halves match, the shape is symmetrical."

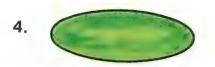
### EXERCISES

Write the numerals 1–8. Next to each numeral write **S** or **NS** to show whether you think each of these shapes is **S**ymmetrical or **N**ot **S**ymmetrical.

















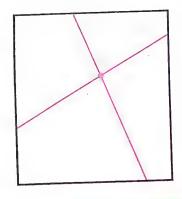
**9.** Do you think symmetrical objects look better than objects which are not symmetrical?

### **Angles**

John drew two lines on a piece of paper. They met in a point. John said, "I have drawn 4 angles."

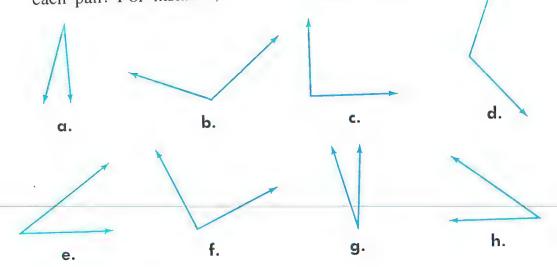
He cut along both lines and fitted the matching edges together.

John said, "The angles fit on top of one another. They are of the same size."

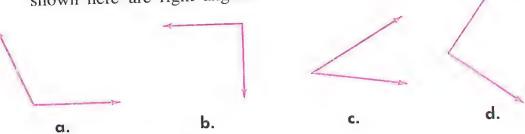


### EXERCISES

1. Pair off the angles here which seem to be the same size. Write the letters to show the angles in each pair. For instance, we would write (c, f).



2. The angle between the corner of a building and level ground is a right angle. Which of the angles shown here are right angles?



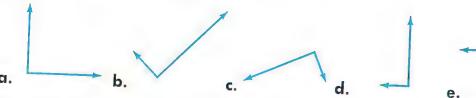
### Right angles

By folding his paper, John made 4 right angles. Do you know how to do this? If you have forgotten, turn back to page 75.

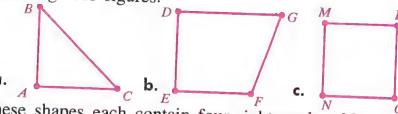


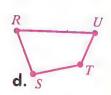
### EXERCISES

1. Are all of these angles the same size? Can you give them a special name?

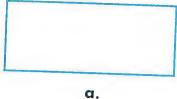


2. Name the right angles shown in each of the following four figures.





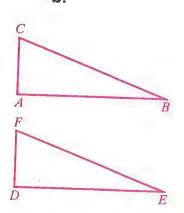
3. These shapes each contain four right angles. Name these shapes.



**4.** Each of the triangles *ABC* and *DEF* has one right angle.

 $m(\overline{AB}) = m(\overline{DE})$  and  $m(\overline{AC}) = m(\overline{DF})$ .

- a. Would DEF fit on top of ABC?
- **b.** Are *ABC* and *DEF* the same size and shape?
- c. ABC and DEF may be fitted together. Name the shape they form.



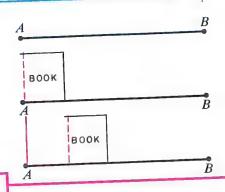
### Parallel lines

If two lines in the same plane do not meet, they do not form an angle. Two lines of this kind are said to be **parallel**.

Sue drew two parallel line segments. First she drew  $\overline{AB}$ .

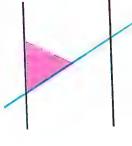
Then she placed one corner of her book on  $\overline{AB}$  and drew a line segment.

Then she moved the book along  $\overline{AB}$  and drew another line segment.

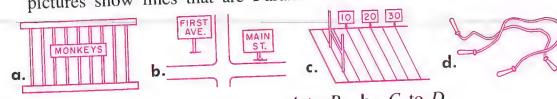


### EXERCISES

1. Use Sue's method to draw two parallel line segments. Draw a line across them, as shown here. How many angles did you form? Are any of the angles of the same size? Cut out the region like the one shown in red, and see if there are other angles of the same size.

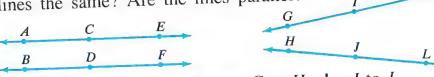


2. Write the letters a to d on your paper. Next to each letter write P or NP to show whether these pictures show lines that are Parallel or Not Parallel.



3. Measure the distance from a. A to B b. C to D

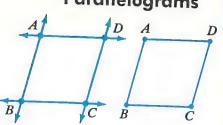
**c.** E to F. Are the distances between the two lines the same? Are the lines parallel?



4. Measure the distance from a. G to H b. I to Jc. K to L. Are the distances the same? Are the lines parallel?

### **Parallelograms**

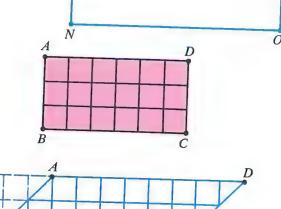
 $\overrightarrow{AB}$  and  $\overrightarrow{CD}$  are parallel.  $\overrightarrow{AD}$  and  $\overrightarrow{BC}$  are parallel.  $\overrightarrow{ABCD}$  is a **parallelogram**.



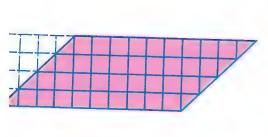
### EXERCISES

M

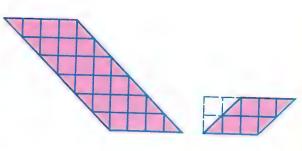
- **1.** Is  $\overline{MN}$  parallel to  $\overline{PO}$ ?
- **2.** Are  $\overline{MP}$  and  $\overline{NO}$  the same length?
- **3.** Are  $\overline{MP}$  and  $\overline{NO}$  parallel?
- 4. Is MNOP a parallelogram?
- **5.** How many unit squares are there in *ABCD*?
- 6. How many unit squares are there in the parallelogram *ABCD*? Remember: You can count two half squares as one square.



- 7. What is the area of each of the regions below?
- 8. What is the name of these shapes?



b.



C.

a.

## Spheres and cylinders





Marvin had a circle on a rubber band.

He spun it so that it looked like a ball.

Another name for a hollow ball is sphere.





Molly had a rectangle on a rubber band.

She spun it so that it looked like a can.

Another name for a hollow can is cylinder.

### EXERCISES

### Answer the questions.

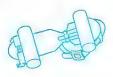
1. Are spheres always the same shape? Are spheres always the same size?

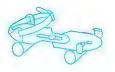






- 2. Can the curved part of a cylinder be made from a flat surface?
- 3. What can you say about the ends of a cylinder?
- **4.** If you roll a sphere, will it roll in any direction?
- 5. If you roll a cylinder, will it roll in any direction?
- 6. What can you say about these roller skates?

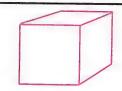




### Spheres and cubes



Michael said, "I have in my hand something which no one has ever seen—the inside of this orange."



This is called a cube.



This is called a sphere.

Without going through the surface can you get inside a cube? inside a sphere?

#### EXERCISES

Use the pictures to answer the questions.

- 1. a. Can the jack-in-the-box get out without opening the cover? b. What shape is the box?
- 2. Can you get the cereal without opening the box?



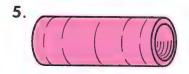
**3.** a. Can you blow up a balloon without having a hole in it? b. What shape is the balloon?

Write the numerals 4–9 on your paper. Next to each numeral write cylinder, cube, or sphere.





7. GAS

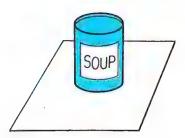


8.

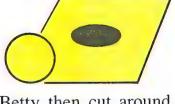


THIS SIDE UP

### Parts of circular regions



1. Betty put a can on her paper. She drew a pencil mark around the base of the can. What shape did she draw?



2. Betty then cut around the figure she had drawn and threw the exterior away. She was left with a circular region.





3. Betty folded her paper to form two parts of the same size. She colored one part. Write the fraction for the colored region.



4. She folded the paper again to form four parts of the same size, and colored three of them.

Write the fraction for

Write the fraction for each colored region.



5.



8



6.



9.



the colored region.

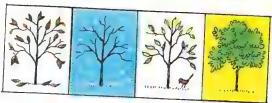
7.



10.



- 1. Write the fraction for the part of the year that is
  - a. spring
  - b. winter and fall.



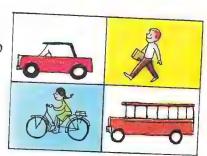
- 2. Write the fraction for the part of the money that goes
  - a. for food
  - b. for food and housing
  - c. for food, clothes and fun.

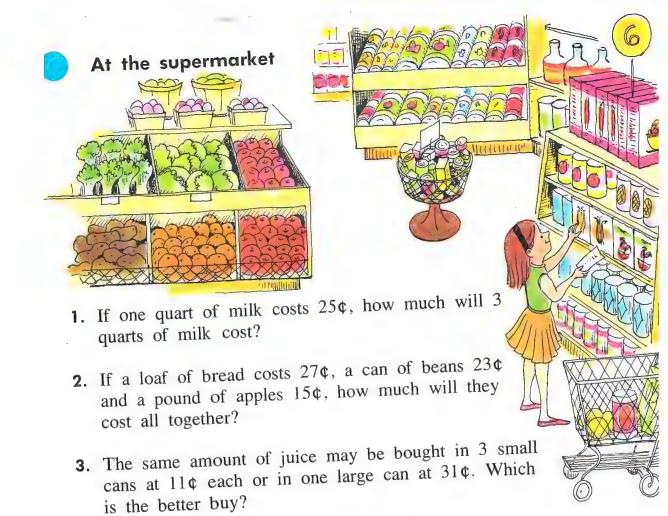


- 3. Write the fraction for the part of the day
  - a. before school and in school
  - b. after school and asleep
  - c. before school, in school, and after school.



- Write the fraction for the children who
  - a. come by bus
  - walk
  - do not walk.





- 4. Cereal can be bought in small 5 ounce boxes for 20¢, or in large 10 ounce boxes for 30¢. Which is the better buy?
- 5. What is the total cost of a pint of ice cream that sells for 39¢ and a pie that sells for 49¢?
- 6. What is the total cost of 4 oranges at 6¢ each and 4 apples at 5¢ each?
- 7. Sarah had \$2.00. She bought fruit for 63¢ and groceries for 87¢. How much does she have left?
- 8. If you had 50¢ and spent 39¢, how much change would you have left?

### Practice page

Complete five other names for the numbers named at the beginning of each row.

- 1. 8:  $6 + \overline{a}$ ,  $12 \overline{c}$ ,  $\overline{e} + 7$ ,  $2 \times \overline{r}$ ,  $\overline{n} \div 6$
- **2.** 12:  $8 + \overline{a}$ ,  $13 \overline{c}$ ,  $\overline{e} 11$ ,  $3 \times \overline{r}$ ,  $\overline{n} \div 3$
- 3. 7:  $3 + \overline{a}$ ,  $13 \overline{c}$ ,  $\overline{e} 12$ ,  $7 \times \overline{r}$ ,  $\overline{n} \div 5$
- **4.** 16:  $12 + \overline{a}$ ,  $24 \overline{c}$ ,  $\overline{e} 14$ ,  $4 \times \overline{r}$ ,  $\overline{n} \div 2$
- 5. 9:  $5 + \overline{a}$ ,  $18 \overline{c}$ ,  $\overline{e} + 7$ ,  $3 \times \overline{r}$ ,  $\overline{n} \div 5$
- **6.** 14:  $9 + \overline{a}$ ,  $15 \overline{c}$ ,  $\overline{e} + 7$ ,  $7 \times \overline{r}$ ,  $\overline{n} \div 2$
- 7. 10:  $9 + \overline{a}$ ,  $10 \overline{c}$ ,  $\overline{e} 12$ ,  $5 \times \overline{r}$ ,  $\overline{n} \div 6$

Name the sums.

- **8.** To  $(6 \times 5)$  add **a.** 3 **b.** 4 **c.** 5 **d.** 2
- **9.** To  $(3 \times 7)$  add **a.** 2 **b.** 4 **c.** 5 **d.** 6
- **10.** To (4 × 8) add **a.** 3 **b.** 6 **c.** 2 **d.** 5
- 11. To  $(3 \times 6)$  add **a.** 4 **b.** 5 **c.** 3 **d.** 1

Name the missing addends.

- **12.** From  $(6 \times 5)$  subtract **a.** 2 **b.** 3 **c.** 4 **d.**
- **13.** From  $(4 \times 6)$  subtract **a.** 3 **b.** 5 **c.** 2 **d.** 4
- **14.** From  $(6 \times 8)$  subtract **a.** 6 **b.** 2 **c.** 4 **d.** 5
- **15.** From  $(7 \times 5)$  subtract **a.** 3 **b.** 2 **c.** 4 **d.** 5

solve the equations.

- **16.**  $(2 \times 3) + (4 \times 5) = \overline{a}$
- **18.**  $(4 \times 5) (3 \times 6) = \overline{c}$
- **20.**  $(7 \times 2) (3 \times 3) = [e]$
- **22.**  $(8 \times 6) (7 \times 5) = \overline{a}$
- **24.**  $(6 \times 9) (5 \times 7) = \overline{n}$
- **26.**  $(5 \times 9) (4 \times 7) = \overline{a}$
- **28.**  $(6 \times 4) + (6 \times 3) = \boxed{c}$
- **30.**  $(4 \times 8) (3 \times 8) = \mathbf{e}$
- **32.**  $(7 \times 3) (3 \times 6) = x$

- 17.  $(6 \times 3) + (2 \times 4) = n$
- **19.**  $(6 \times 9) + (7 \times 8) = 0$
- **21.**  $(7 \times 3) + (6 \times 2) = \mathbf{r}$ 
  - **23.**  $(8 \times 5) + (4 \times 3) = \boxed{c}$
  - **25.**  $(9 \times 6) (6 \times 5) = 0$
  - **27.**  $(9 \times 5) (4 \times 4) = \mathbf{r}$
  - **29.**  $(3 \times 4) + (6 \times 2) = x$
  - **31.**  $(7 \times 7) + (7 \times 7) = \boxed{z}$
  - **33.**  $(8 \times 3) (5 \times 4) = \boxed{s}$

### Number pairs

In the third grade classroom there are thirty desks. They are arranged like this. The name on top of each desk shows who sits there.

Caci	I desk blie iis	,,,,,,		
Bob	Karen	Michael (3, 6)	Bill	Mark
Mary	Ellen	Tommy	Fred	Patty
Sharon	Susan	John	Carol	Jack
Pat (1, 3)	Peter	Judy	Eric	Karen
Carol	Edie	Jane	Keith	Rick
David	Karen	Eddie	John	Lynne
		Front		

There are 6 rows of desks. There are 5 columns of desks from front to back. Pat is sitting in the first column from the door and the third row from the front. We label Pat's desk (1,3). Michael's desk is in the third column from the door and the sixth row from the front. We label Michael's desk (3,6).

### EXERCISES

Write number pairs to label

- 1. Jack's desk
- 3. Mark's desk

- 2. Fred's desk
- 4. Lynne's desk

There are two Carols, three Karens, two Johns. Write number pairs to label the desks of

- 5. the two Carols
- 6. the three Karens
- 7. the two Johns

Name the child sitting at

- **8.** (4, 5)
- **9**. (4, 6)
- **10**. (4, 2)
- **11.** (4, 1)
- **12**. (1, 4)

- **13**. (2, 4)
- **14**. (3, 4)
- **15.** (4, 4)
- **16.** (5, 4)
- **17.** (5, 6)

6

Sixth Street				J(4,	6)		
Fifth Street	В		N		R		
Fourth Street	M		C		E		
Third Street		G		D			
Second Street	A(1)	, 2)	L		K		,
First Street							
This bucce		ıne	Ð	ne	4)	4)	
	Avenue	Second Avenue	Third Avenue	Fourth Avenue	Avenue	Sixth Avenue	
	Ave	y pı	Av	h A	Ave	Av	
·	First	ecor	hird	ourt	Fifth	ixth	
		S		IT.		S	

We see that point A is at the corner of First Avenue and Second Street. We may label that point (1,2). Point J is at the corner of Fourth Avenue and Sixth Street. We may label that point (4,6). Notice that we name the Avenue first, then the Street.

#### EXERCISES

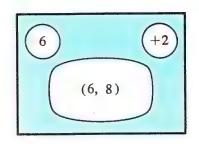
- 1. What are the points that are labeled
  - a. (2,3)
- **b.** (5, 4)
- (5,5)
- **d.** (4, 3)

- 2. Write a number pair to label the points
  - $\mathbf{a}.$  B
- **b**. *K*
- c. L
- **d.** *C*
- e. *M*

- **f.** N
- g. D
- h. E
- i. G
- i. *R*
- **3.** a. Bob walked from (3, 5) to (3, 2). How many blocks was that?
  - **b.** How many blocks from (6, 2) to (2, 2)?
  - **c.** How many blocks across town from (3, 3) to (5, 6)? How many blocks up town?
  - **d.** How many blocks across town from (1, 6) to (4, 2)? How many blocks down town?

### **Number pairs**

We have used our TV set to complete number pairs. For example, if the number knob shows 6 and the tuned in rule is "add 2", the screen would show the number pair (6,8).



#### EXERCISES

Think of an addition or subtraction rule, then complete the sets.

- 1.  $\{(2,4), (3,5), (6, \mathbf{a}), (7, \mathbf{c}), (8, \mathbf{e}), (5, \mathbf{n})\}$
- **2.**  $\{(1,5), (3,7), (6, \boxed{0}), (2, \boxed{r}), (5, \boxed{s}), (8, \boxed{u})\}$
- **3.**  $\{(2,8), (3,9), (4, \overline{r}), (7, \overline{x}), (9, \overline{z}), (12, \overline{a})\}$
- **4.**  $\{(8,4), (6,2), (9, \mathbf{c}), (5, \mathbf{e}), (4, \mathbf{n}), (7, \mathbf{o})\}$
- **5.**  $\{(10,5), (11,6), (7, \mathbf{n}), (8, \mathbf{s}), (9, \mathbf{u}), (5, \mathbf{x})\}$

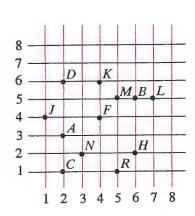
Think of a multiplication or division rule, then complete the sets.

- **6.**  $\{(2,6), (4,12), (5, \mathbf{a}), (6, \mathbf{c}), (7, \mathbf{e}), (9, \mathbf{n})\}$
- 7.  $\{(3,12), (5,20), (6, \mathbf{e}), (2, \mathbf{r}), (8, \mathbf{s}), (7, \mathbf{u})\}$
- **8.**  $\{(6,3), (8,4), (10, \overline{v}), (12, \overline{x}), (4, \overline{z}), (18, \overline{a})\}$
- **9.**  $\{(1,5), (3,15), (4, \mathbf{C}), (2, \mathbf{E}), (6, \mathbf{M}), (8, \mathbf{O})\}$

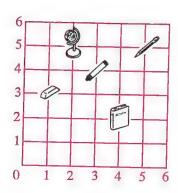
Study the chart and answer the questions.

The letter N labels the point where 3 red and 2 black meet. We shall label it (3,2).

- **10.** Write number pairs for the point labeled
  - a. B b. D c. F d. H e. J f. A
- 11. Name the points which are labeled by these number pairs:
  - **a.** (2,3) **b.** (4,4) **c.** (6,2)
  - **d.** (5,5) **e.** (2,6) **f.** (7,5)





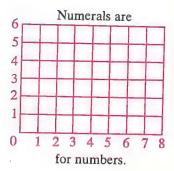


1. Write the words pencil, eraser, book, crayon, and globe on your paper.

Next to each word write its number pair.

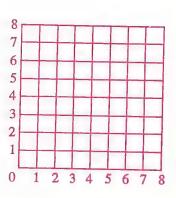
Copy the chart at the right. Then write the letter

- **2.** N on the point (3, 6)
- 3. S on the point (7, 2)
- **4.** M on the point (5,4)
- **5.** E on the point (6,3) and
- **6.** A on the point (4,5)
- 7. Does the chart make a true statement?



Copy the chart at the right.

- **8.** Use the chart to form a letter of the alphabet by starting at the number pair (1,2) and moving in order to (2,6), (4,3), (6,6), (7,2).
- 9. Use the chart to form a one-digit numeral by starting at the number pair (2,7) and moving in order to (4,7), (5,7), (4,5), (3,3).





Michael was in charge of the temperature records. Every day at 10 A.M. Michael read the thermometer and wrote his reading in his notebook.

MAY 5: 56

MAY 6: 58

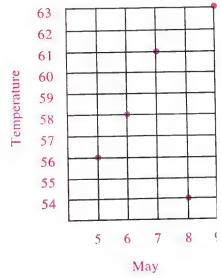
MAY 7: 61

MAY 8: 54

MAY 9: 63

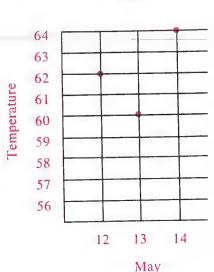
Michael could keep his records using number pairs and a chart. His records would look like the chart on the right.

- 1. From the chart complete the number pairs,  $(5, \overline{a})$ ,  $(6, \overline{c})$ ,  $(7, \overline{e})$ ,  $(8, \overline{n})$ .
- 2. Which was the hottest day? Which was the coldest?



A chart like Michael's is called a **graph**. Here is a graph which shows the temperature for five other days in May.

- 3. What was the temperature on May 12? May 13? May 15?
- **4.** From the graph complete the number pairs  $(12, a_0), (13, c_0), (14, e_0), (15, n_0)$ .
- 5. On which day was the temperature 60°?



### More on graphs



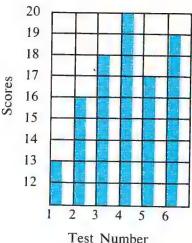
Use the graphs to answer the questions.

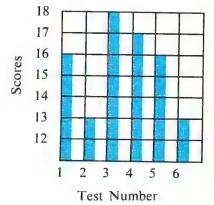
Keith made a graph of his scores for six tests in mathematics. They were: 13, 16, 18, 20, 17, 19.

1. Would you say that Keith's scores are getting better?

Here are Judy's scores on six spelling tests. Read the graph to answer the questions.

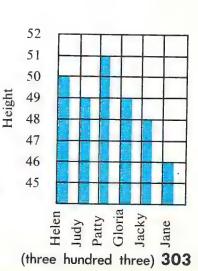
- 2. What did Judy score
  - a. on the first test?
  - **b.** on the sixth test?
- 3. What was Judy's highest score?
- 4. On which tests was Judy's score the same?





Helen's teacher made a graph of the height in inches of six girls in the class.

- **5.** Who is tallest?
- **6.** Who is shortest?
- 7. Name two girls who are the same height.





Use the graphs to answer the questions.

Sally had a birthday party. She invited six friends. At the party, they played a game to see who could eat the most cookies.

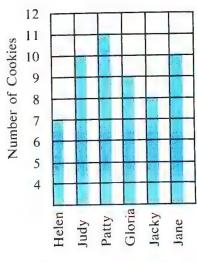
- 1. Who won the contest?
- 2. Name the two friends who ate the same number of cookies.
- 3. Who ate the least number of cookies?

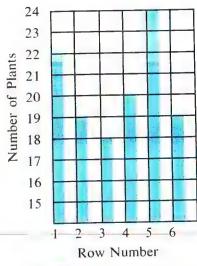
There are 6 rows of children in Barbie's class. Each row planted seeds in a box. After some weeks they counted the number of plants which grew.

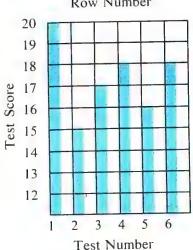
- **4.** How many plants grew in the fourth row's box?
- 5. Which row grew the most plants?
- 6. Which row grew the fewest plants?
- 7. Which rows grew 19 plants?

Here are Rick's scores in six spelling tests.

- **8.** What was Rick's score on the first test?
- 9. What was Rick's highest score?
- 10. What was Rick's lowest score?
- 11. On which test did Rick score 18?







### Using sets and numbers



1. Four boys caught 36 fish. They wanted to put the same number in four buckets. How many should go into each bucket?



- 2. There were 97 children at the picnic. Twelve more came. How many children were there in all?
- 3. How many stamps are on the sheet on the right? Can you use a number pair to locate the stamp in red?
- 1. Julie's scores in her spelling tests were 85, 76 and 92. Bill had scores of 91, 79, 86. Who had the better total score?



How much money is there in all if the large bank has 300 pennies, the next largest 157 pennies, the next 93 pennies and the smallest 22 pennies?



Claire and her mother ate at a restaurant. How much money did Mother have to pay for the meal?

Mr. Boyle spent \$3.45 at the restaurant. How much change should he receive if he gives the waiter \$5.00?

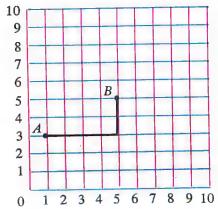


(three hundred five) 305

### Lattices

This is a **lattice**. It is made up of number lines. We have colored one set of number lines red, the other blue. We shall show moves along the blue lines like this  $\square$ , or this  $\triangleleft$ , and moves along the red lines like this  $\square$ , or this  $\square$ . The arrow tells you in which direction to move.

If we start at the point (1,3), which is labeled A and move A and D, we stop at (5,5), which is labeled B.



### EXERCISES

Write the number pairs to complete this chart.

		,	-
	Start	Move	Stop
1.	(1, 3)	2>	a
3.	(2, 6)	5>	c
5.	(3, 6)	4>	e
7.	(2,4)	6>	[n]

	Start	Move	Stop
2.	(1, 3)	4	S
4.	(2, 4)	(5)	u
6.	(3, 5)	2	v
8.	(4, 6)	3	[x]

Write the number pairs to complete this chart.

	Start					
	(2,4)			a		
	(3, 4)			c		
13.	(4, 1)	5>	6	e		
15.	(6, 7)	3>	$\hat{1}$	$\boldsymbol{n}$		459
					•	

Start
 Move
 Stop

 10. 
$$(2,5)$$
 6>  $\boxed{4}$ 
 $\boxed{s}$ 

 12.  $(7,8)$ 
 $\boxed{3}$ 
 $\boxed{6}$ 
 $\boxed{u}$ 

 14.  $(8,5)$ 
 $\boxed{4}$ 
 $\boxed{4}$ 
 $\boxed{v}$ 

# **14.** (8, 5) €4 ⊈ Ū **16.** (6, 1) €2 3 x

#### EXTRA FOR EXPERTS

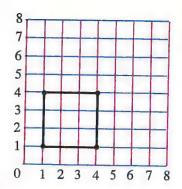
Write the number pairs to complete this chart.

,					-				~ .
	Start	Mo	ve	Stop		Start	Mo	ove	Stop
	a				18.	0	€2	2	(3, 6)
			•	(7, 5)					(5, 2)
			•	(9, 8)	22.	S	<b>4</b> 4	3	(4, 7)

#### Lattices

We start at (1, 1), move  $\overline{3}$  to (1, 4), We form a square.

We can write this pattern like this: Start Move Stop (3); (3)∗; (3); (3) (1, 1)(1, 1)



### EXERCISES

Copy the lattice on your paper and find the answers.

1. Form the letter P on the lattice by starting at (1,1) and making the moves: 6; 3; 2; 43. What letters are formed by the following moves:

Start Move

- (6, 8)**6**; **3**→; **1**
- [5]; [3]→; [Ŝ] (1,7)
- (5, 6)4; 4°; 2°; «1

Vrite number pairs to complete the chart.

#### Start Move Stop

- **2.** (4, 3) 2 |a|
- |c|
- **6.** (2, 9) **6**→ e

- Start Move Stop
- 3. (4,3)  $\boxed{3}$ 2 |n|
- **5.** (6, 3) **←**4 2 0
- **7.** (2, 9) 4 6> r

/rite number pairs to complete the chart.

	Start	Move	Stop
0		[2]	

8. (2,4) (3) |a|10. (6, 1) 7 7 |c|

- Start Move Stop
- **9.** (3, 7) 4 4 e**11.** (0, 0) 5 ÷ +5 |n|
- 12. If you move 2 and 2 will you stop where you started?
- 13. What can you say about the moves in these exercises?

### LOOKING WAY BACK

Complete these equations.

1. 
$$3 \times 4 = 12$$
,  $1 \times 4 = 4$ , so  $4 \times 4 = \mathbf{a}$  and  $16 \div 4 = \mathbf{c}$ 

**2.** 
$$5 \times 6 = 30$$
,  $2 \times 6 = 12$ , so  $7 \times 6 = \mathbf{e}$  and  $42 \div 6 = \mathbf{n}$ 

**3.** 
$$6 \times 4 = 24$$
,  $4 \times 6 = 0$ , so  $24 \div 6 = r$  and  $24 \div 4 = s$ 

**4.** 
$$6 \times 9 = 54$$
,  $9 \times 6 = \overline{u}$ , so  $54 \div 6 = \overline{v}$  and  $54 \div 9 = \overline{x}$ 

Write four equations for each of these sets of two factors and a product.

Write the simplest numeral for

In 9763 what is

Solve the equations.

15. 
$$56 + 27 = a$$

16. 
$$64 - 28 = \boxed{c}$$

17. 
$$44 - 27 = e$$

18. 
$$42 \div 7 = 0$$

19. 
$$9 \times 6 = r$$

**20.** 
$$72 \div 8 = s$$

Name the sums.

Name the numbers.

27. Name the prime numbers among these: {7, 9, 11, 13, 15, 17}

28. Name two factors of 42, other than 42 and 1.

29. Name three factors of 24, other than 24 and 1.

Complete the equations.

**30.** 
$$2 \times \bar{a} = 4$$
,

$$c \times 2 = 4$$

$$4 \div 2 = \mathbf{e},$$

$$\overline{c} \times 2 = 4, \quad 4 \div 2 = \overline{e}, \quad \frac{1}{2} \text{ of } 4 = \overline{n}$$

**31.** 
$$3 \times \boxed{0} = 6$$
,  $\boxed{r} \times 2 = 6$ ,  $6 \div 3 = \boxed{s}$ ,  $\frac{1}{2}$  of  $6 = \boxed{u}$ 

$$\mathbf{r} \times 2 = 6$$

$$6 \div 3 = \mathbf{S},$$

$$\frac{1}{2}$$
 of  $6 = \overline{u}$ 

308 (three hundred eight)

Write the numerals 1-10. Next to each write the letters from a to j, matching the names on the left with the drawings on the right.

1. cube
2. parallel lines
3. sphere
4. rectangle
5. closed curve

c. h.

6. square7. cylinder

8. curve9. circle

10. parallelogram

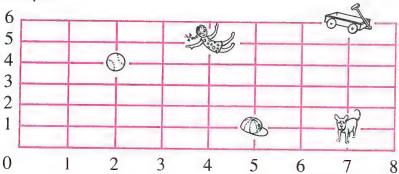
e. 55 i.

Write the fraction for the part of each pie that has been eaten.



15. Write the fractions for the parts of the pies that remain.

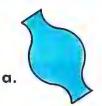
Write the words baseball, hat, wagon, doll, dog on your paper. Next to each word write the number pair which locates the object.



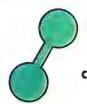
### LOOKING BACK

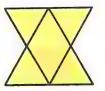
Answer the questions.

1. Which of the figures below are symmetrical?





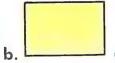




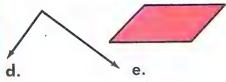


2. Write the name of each figure below.



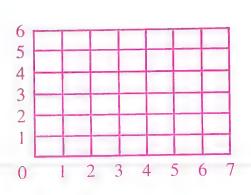




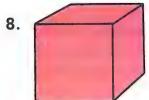


Copy the chart and write the letter:

- **3.** P on the point (3,3) and (2,4);
- **4.** E on the point (5,1);
- 5. A on the point (1,5);
- **6.** L on the point (4,2);
- 7. What word did you spell?



Which of the following pictures illustrate right angles?



9.



10.

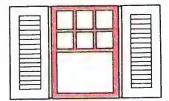


11.





13.

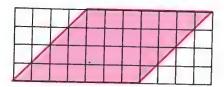


### EXTRA FOR EXPERTS

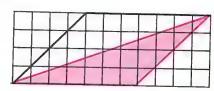


1. Name the number of unit squares contained in these regions.

a.

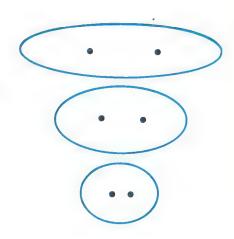


b.



- 2. Make graphs of your test scores like those on page 302 or those on page 303.
- 3. Find other graphs like the ones we have been using. Look in newspapers or magazines.

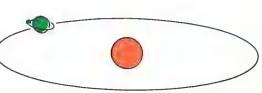
- **4. a.** Each of the figures on the right is called an **ellipse**. Is an ellipse a closed curve?
  - **b.** What can you say about the shape of an ellipse from these drawings?



5. a. The path the earth follows as it moves around the sun is an \_\_\_? \_\_. Do you know of any other examples of ellipses?

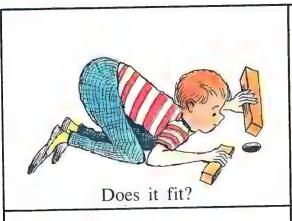
 $\mathcal{H}$ 

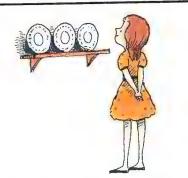
**b.** A circle fits inside a square: An ellipse fits inside a \_?\_.



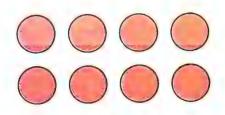
## CHAPTER 11 Addition, Subtraction, Multiplication, Division

### Thinking about the set





Can she get four plates?



Can we form sets of 3 without any objects left over?

 $\{0, 2, 4, 6, 8, 10, 12, \ldots\}$ 

$$2 + 2 = 4$$
  $4 + 8 = 12$   
 $2 + 4 = 6$   $6 + 4 = 10$ 

Can you name two even numbers whose sum is an odd number?

### EXERCISES

- 1. With the numbers we have been using so far, can we subtract 9 from 6, 24 from 10, 17 from 8?
- 2. If the missing addend has to be a counting number, can we solve the equation  $\boxed{a} + 3 = 2$ ?
- 3. If the missing factor has to be a counting number, can we solve the equation  $3 \times \boxed{e} = 4$ ?
- **4.** If we add two odd numbers, will the sum be another odd number?
- 5. If we multiply two prime numbers, will the product be another prime number?

### The order rule



It is important to wait for the car to stop before you get out. Order is important here.

#### EXERCISES

Write the numerals of the examples where order is mportant.

- 1. Taking off your socks and shoes.
- 3. Opening a door and going through the doorway.
- 5. Eating bread and eating meat.
- **7.** Adding 4 and 3.

- 2. Filling a bath tub, taking a bath.
- 4. Putting an apple in your mouth, biting the apple.
- 6. Turning on TV, watching a show on TV.
- **8.** Multiplying 3 and 6.

olve these equations.

9. 
$$4+3=3+r$$

10. 
$$5+4=4+\sqrt{a}$$

9. 
$$4+3=3+r$$
 10.  $5+4=4+a$  11.  $17+1=1+n$ 

12. 
$$64 + 3 = |\overline{c}| + 64$$

13. 
$$6 + 19 = \boxed{0} + 6$$

**12.** 
$$64 + 3 = \boxed{c} + 64$$
 **13.**  $6 + 19 = \boxed{o} + 6$  **14.**  $27 + \boxed{x} = 3 + 27$ 

an you state the general rule?

15. In addition the order of the \_\_? does not change the sum.

olve these equations.

**16.** 
$$3 \times 4 = 4 \times \overline{a}$$

**16.** 
$$3 \times 4 = 4 \times a$$
 **17.**  $3 \times c = 6 \times 3$  **18.**  $8 \times 7 = e \times 8$ 

18. 
$$8 \times 7 = e \times 8$$

19. 
$$3 \times 49 = 49 \times n$$

**20.** 
$$8 \times 76 = 0 \times 8$$

**19.** 
$$3 \times 49 = 49 \times \mathbf{m}$$
 **20.**  $8 \times 76 = \mathbf{o} \times 8$  **21.**  $601 \times 3 = 3 \times \mathbf{r}$ 

an you state the general rule?

**22.** In multiplication, the order of the \_\_\_? does not change the \_\_?\_.

### The grouping rule

#### In addition:

$$45 + 3 = (40 + 5) + 3$$

$$= 40 + (5 + 3)$$

$$= 40 + 8$$

$$= 48$$
so  $45 + 3 = 48$ 

#### In multiplication:

$$2 \times 40 = 2 \times (4 \times 10) 
= (2 \times 4) \times 10 
= 8 \times 10 
= 80 
so 2 \times 40 = 80$$

#### EXERCISES

Solve these equations.

1. 
$$(2+2)+1=2+(2+1)$$
, so  $\overline{a}+1=2+\overline{n}=\overline{c}$ 

so 
$$[a] + 1 = 2 + [n] = [a]$$

**2.** 
$$(2+3)+4=2+(3+4)$$
,

so 
$$o + 4 = 2 + e = s$$

3. 
$$(40 + 7) + 9 = 40 + (7 + 9)$$
, so  $47 + 9 = 40 + x = z$ 

so 
$$47 + 9 = 40 + \mathbf{x} = \mathbf{z}$$

**4.** 
$$40 + (10 + 3) = (40 + 10) + 3$$
, so  $40 + 13 = 50 + \boxed{e} = \boxed{c}$ 

$$80 \ 40 + 13 = 50 + \boxed{e} = \boxed{c}$$

Can you state the general rule?

5. The way the \_\_? are grouped for addition does not change the sum.

Solve these equations.

**6.** 
$$3 \times (2 \times 10) = (3 \times 2) \times 10$$
, so  $3 \times 20 = (3 \times \overline{a}) \times 10 = \overline{n}$ 

**7.** 
$$4 \times (3 \times 10) = (4 \times 3) \times 10$$
, so  $4 \times 30 = (4 \times \mathbf{c}) \times 10 = \mathbf{0}$ 

**8.** 
$$2 \times (3 \times 10) = (2 \times 3) \times 10$$
, so  $2 \times 30 = (2 \times \overline{e}) \times 10 = \overline{u}$ 

**9.** 
$$2 \times (2 \times 10) = (2 \times 2) \times 10$$
, so  $2 \times 20 = (2 \times 5) \times 10 = \overline{r}$ 

Can you state the general rule?

10. The way the \_\_?\_ are grouped for multiplication does not change the \_\_?\_\_.

Use the grouping rule to name these products or sums.

11. 
$$3 \times 2 \times 5$$

13. 
$$8 \times 2 \times 5$$
 14.  $2 \times 2 \times 3$ 

**15.** 
$$10 + 8 + 4$$
 **16.**  $4 \times 2 \times 10$ 

16. 
$$4 \times 2 \times 10$$

Study this example and then name the sums.

$$27 + 5 = (20 + 7) + 5 = 20 + (7 + 5) = 20 + 12 = 32$$

$$21.54 + 9$$

### Using the order and grouping rules



Study these methods to name the sum 27 + 14.

#### **Equation** method 27 + 14 = 27 + (10 + 4)= (27 + 10) + 4= 37 + 4 41

### Short method 1 . 27 + 1441

#### EXERCISES

Jse the equation method to name these sums.

$$1. 27 + 45$$

$$3.38 + 24$$

11. 
$$82 + 19$$

13-24. Use the short method to name the sums in exercises 1-12.

study these equations.

$$43 + 24 = (40 + 3) + (20 + 4)$$

$$= 40 + 20 + 3 + 4$$

$$= (40 + 20) + (3 + 4)$$

$$= 60 + 7$$

$$= 67$$

Copy and complete these equations.

25. 
$$36 + 23 = (30 + 6) + (20 + 3)$$
  
=  $30 + 20 + \boxed{a} + 3$   
=  $(30 + \boxed{n}) + (\boxed{c} + \boxed{o})$   
=  $\boxed{e} + \boxed{r}$   
=  $\boxed{s}$ 

26. 
$$87 + 11 = (80 + 7) + (10 + 1)$$
  
=  $80 + 10 + 7 + 1$   
=  $(80 + 10) + (\overline{u} + \overline{v})$   
=  $\overline{x} + \overline{z}$   
=  $\overline{a}$ 



### One and zero are important numbers

Solve these equations.

1. 
$$8 \times 1 = a$$

**2.** 
$$1 \times 12 = \bar{c}$$

1. 
$$8 \times 1 = a$$
 2.  $1 \times 12 = c$  3.  $17 \times 1 = e$ 

4. 
$$8 \times n = 8$$

5. 
$$9 \times 0 = 9$$

**4.** 
$$8 \times \overline{n} = 8$$
 **5.**  $9 \times \overline{o} = 9$  **6.**  $300 = 300 \times \overline{r}$ 

7. 
$$7 \div 1 = s$$

8. 
$$18 \div 1 = u$$

**7.** 
$$7 \div 1 = \boxed{s}$$
 **8.**  $18 \div 1 = \boxed{u}$  **9.**  $29 \div 1 = \boxed{v}$ 

10. 
$$9 \div x = 9$$

11. 
$$16 \div \mathbf{z} = 16$$

10. 
$$9 \div \boxed{x} = 9$$
 11.  $16 \div \boxed{z} = 16$  12.  $489 \div \boxed{a} = 489$ 

Name these products.

Name these missing factors.

Can you state the general rule?

25. If one of the factors is one, the product is the

Name these sums.

Name these sums or missing addends.

Can you state the general rule?

# Add and subtract, multiply and divide



Study these equations.

$$5 + 3 = 8$$
,  $8 - 3 = 5$ 

To add 3 and then subtract 3 is the same as adding 0.

$$4 \times 3 = 12, \quad 12 \div 3 = 4$$

To multiply by 3 and then divide by 3 is the same as multiplying by 1.

#### EXERCISES

Solve these equations.

1. 
$$4 \times 6 = 24$$
, so  $24 \div 6 = [a]$ 

$$a \times 6 - 24$$
, so  $24 \div 6 = a$ 

**3.** 
$$9 \times 7 = 63$$
, so  $63 \div 7 = \overline{c}$ 

5. 
$$8 \times 9 = 72$$
, so  $72 \div 9 = [e]$ 

**7.** 
$$8 \times 5 = 40$$
, so  $40 \div 5 = \mathbf{n}$  **8.**  $7 \times 3 = 21$ , so  $21 \div 3 = \mathbf{x}$ 

**2.** 
$$7 \times 8 = 56$$
, so  $56 \div 8 = \boxed{0}$ 

**3.** 
$$9 \times 7 = 63$$
, so  $63 \div 7 = \boxed{c}$  **4.**  $8 \times 4 = 32$ , so  $32 \div 4 = \boxed{r}$ 

**5.** 
$$8 \times 9 = 72$$
, so  $72 \div 9 = \mathbf{e}$  **6.**  $4 \times 6 = 24$ , so  $24 \div 6 = \mathbf{s}$ 

8. 
$$7 \times 3 = 21$$
, so  $21 \div 3 = \bar{x}$ 

Name the sum or missing addend for each of these pairs of examples.

15

6)42

Name the product or missing factor for each of these pairs of examples.

Solve these equations.

**15.** 
$$(9+7)-7=n$$

**16.** 
$$(6+4)-4=\boxed{c}$$

17. 
$$(8+3)-3=0$$

18. 
$$(27 + 3) - 3 = \mathbf{r}$$

19. 
$$(18+6)-6=$$
 **S**

**20.** 
$$(21 + 5) - 5 = \overline{u}$$

**21.** 
$$(5 \times 6) \div 6 = x$$

**22.** 
$$(6 \times 8) \div 8 = \mathbf{Z}$$

**23.** 
$$(7 \times 9) \div 9 = \mathbf{a}$$

**24.** 
$$(8 \times 3) \div 3 = n$$

**25.** 
$$(4 \times 6) \div 6 = \mathbf{0}$$

**26.** 
$$(3 \times 9) \div 9 = e$$

**27.** 
$$(7-3)+3=v$$

**28.** 
$$(9-5)+5=c$$

**29.** 
$$(7-2)+2=u$$

**30.** 
$$(12-3)+3=\overline{r}$$

$$31. (14-6)+6=5$$

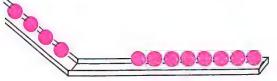
**31.** 
$$(14-6)+6=5$$
 **32.**  $(15-2)+2=x$ 

**33.** 
$$(21 \div 7) \times 7 = a$$

31. 
$$(14-6)+6=3$$
  
34.  $(12 \div 6) \times 6 = \boxed{e}$ 

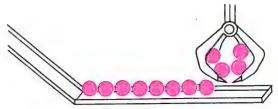
**35.** 
$$(20 \div 4) \times 4 = \boxed{c}$$

# Doing and undoing



Making a set of 12 from a set of 8 and a set of 4.

$$8 + 4 = \boxed{a}$$



Making a set of 4 and a set of 8 from a set of 12.

$$12 - 4 = e$$

#### EXERCISES

Solve the equations.

1. 
$$8 + 4 = 12$$
, so  $12 - 4 = \mathbf{a}$ 

**2.** 
$$64 + 17 = 81$$
, so  $81 - 17 = \boxed{c}$ 

3. 
$$999 + 9 = 1008$$
, so  $1008 - 9 = \boxed{e}$ 

**4.** 
$$308 + 4 = 312$$
, so  $312 - 4 = \mathbf{m}$ 

**5.** 
$$3946 + 14 = 3960$$
, so  $3960 - 14 = \mathbf{0}$ 

**6.** 
$$64,683 + 17 = 64,700$$
, so  $64,700 - 17 = \boxed{r}$ 

7. 
$$487 + 42 = 529$$
, so  $529 - 42 = \overline{u}$ 

**8.** 
$$1808 + 39 = 1847$$
, so  $1847 - 39 = \boxed{r}$ 

Solve the equations.

9. 
$$(7+5)-5=7+a=s$$

**10.** 
$$(12+6)-6=12+\mathbf{c}=\mathbf{u}$$

11. 
$$(6-6) + 8 = \mathbf{e} + 8 = \mathbf{v}$$

**12.** 
$$(15-15) + 10 = \mathbf{m} + 10 = \mathbf{x}$$

**14.** 
$$(26+3) - \mathbf{r} = 26 + 0 = \mathbf{s}$$

**15.** 
$$(20 + 3) - 3 = 20 + \overline{a} = \overline{e}$$

**16.** 
$$(8+8) - 8 = 8 + \mathbf{n} = \mathbf{c}$$

**17.** 
$$36 + (\underline{r} - 4) = 36 + 0 = \underline{u}$$

**18.** 
$$41 + (3 - \overline{v}) = 41 + 0 = \overline{x}$$

**19.** 
$$80 + (9 - 9) = 80 + \mathbf{z} = \mathbf{0}$$

**20.** 
$$(a + 7) - 7 = e + 0 = 29$$

Can you state the general rule?

22. To add a number and then subtract that number is the same as adding 
$$[e]$$
, for example  $(6 + 4) - 4 = 6 + [c]$ .

318 (three hundred eighteen)

# Checking addition and subtraction



Sue uses subtraction to check her addition.

69

To check the addition 
$$+\frac{26}{69}$$
 she subtracts  $-\frac{26}{69}$ .

When she names the missing addend 43, she knows her work is correct.

# EXERCISES

Name the sums and check your additions by subtraction.

1. 4. 4 6 6

Bobby uses addition to check his subtraction.

26

To check the subtraction 
$$-\frac{26}{17}$$
 he adds  $+\frac{17}{17}$ .

When he names the sum 43, he knows his work is correct.

Name the missing addend and check your subtraction by addition.

# Adding hundreds, tens, and ones

Study these methods of naming the sum 428 + 944.

Equations				
8	+	4	=	12
20	+	40	=	60
400	+	900	=	1300
428	+	944	=	1372

# Number pattern

8	4	12
20	40	60
400	900	1300
428	944	1372

# Short method

$$428 + 944 \over 1372$$

#### EXERCISES

Write the numerals to solve these equations and complete these number patterns.

1. 
$$9 + 4 = a$$
  
 $30 + 50 = c$   
 $600 + 800 = e$   
 $n + o = r$ 

2. 
$$6 + 8 = \boxed{s}$$
  
 $20 + 40 = \boxed{v}$   
 $500 + 900 = \boxed{x}$   
 $\boxed{c} + \boxed{a} = \boxed{e}$ 

8	+	1	=	c
40	+	80	=	n
300	+	400	=	0
v	+	u	=	x

4.	8	3	v
	20	50	e
	300	600	n
	u	r	S

Name the sums.

3

- 12. If a farmer has 192 hens in one hen house and 457 in another, how many hens does he have?
- 13. If a farmer has 382 sheep in one field and 245 in another, how many sheep does he have?
- **14.** If a farmer has 271 pigs in one pen and 148 pigs in another, how many pigs does he have?

320 (three hundred twenty)

# Adding thousands, hundreds, tens, and ones



Study these methods of naming the sum 5263 + 3925.

Equations					
3	+	5	=	8	
60	+	20	=	80	
200	+	900	=	1100	
5000	+	3000	=	8000	
5263	+	3925	=	9188	

3	5	8
60	20	80
200	900	1100
5000	3000.	8000
5263	3925	9188

Number pattern

Sho	ort method
	5263
	+ 3925
	9188

# EXERCISES

Vrite the numerals to solve these equations.

1. 
$$3 + 6 = \boxed{a}$$
  
 $20 + 30 = \boxed{c}$   
 $300 + 500 = \boxed{e}$   
 $1000 + 3000 = \boxed{n}$   
 $\boxed{o} + \boxed{r} = \boxed{s}$ 

2. 
$$4 + 7 = \mathbf{u}$$
  
 $40 + 40 = \mathbf{v}$   
 $400 + 200 = \mathbf{x}$   
 $5000 + 4000 = \mathbf{z}$   
 $\mathbf{a} + \mathbf{c} = \mathbf{e}$ 

3. 
$$6 + 3 = a$$
  
 $90 + 30 = c$   
 $700 + 100 = e$   
 $4000 + 3000 = n$   
 $0 + s = r$ 

lame the sums.

- 14. How many people in all live in the towns of Smiley and Friendly?
- 15. How many people in all live in the towns of Smiley and Happy?
- 16. How many people in all live in the towns of Happy and Friendly?



# Multiplying hundreds, tens, and ones

Study these methods of naming the product  $3 \times 243$ .

Equation method $3 \times 243 = 3 \times (200 + 40 + 3)$ $= (3 \times 200) + (3 \times 40) + (3 \times 3)$ $= 600 + 120 + 9$ $= 729$	Four-step method  243  ×3  9  120  600  729	Short method  243  × 3  729
--	---	-----------------------------

#### EXERCISES

Solve these equations.

1. 
$$3 \times 281 = 3 \times (200 + 80 + 1)$$
  
=  $(3 \times 200) + (3 \times 80) + (3 \times 1)$  / 5 3  $\%$   
=  $\boxed{a}$  +  $\boxed{c}$  +  $\boxed{e}$ 

2. 
$$4 \times 162 = 4 \times (100 + 60 + 2)$$
  
=  $(4 \times 100) + (4 \times 60) + (4 \times 2)$   
=  $\boxed{a} + \boxed{c} + \boxed{e}$   
=  $\boxed{m}$ 

Name these products using the 4-step method.

7. 
$$7 \times 141 = r$$

5. 
$$5 \times 171 = e$$
8.  $8 \times 121 = s$ 

Name these products using the short method.

15. 273 16. 462 17. 463 18. 182 19. 284 
$$\times$$
 2  $\times$  2

Solve the problem.

21. During the summer 231 windows in the school building were washed. If each window has 4 panes, how many window panes were washed?

**6.**  $5 \times 141 = 0$ 

# Multiplying hundreds, tens, and ones



Study these methods of naming the product  $6 \times 243$ .

Number pattern	Four-step method	Short method
×       6         3       18         40       240         200       1200         243       1458	$ \begin{array}{r} 243 \\ \times 6 \\ \hline 18 \\ 240 \\ \underline{1200} \\ 1458 \end{array} $	21 243 × 6 1458

# EXERCISES

Complete these number patterns.

$$\begin{array}{c|cccc}
\mathbf{2.} & \times & 6 \\
\hline
 & 7 & \mathbf{r} \\
\hline
 & 10 & \mathbf{s} \\
\hline
 & 300 & \mathbf{u} \\
 & \mathbf{v} & \mathbf{z}
\end{array}$$

Name the products using the 4-step method.

- **5.**  $4 \times 612$
- 6. 6 × 315
- **7.**  $7 \times 412$
- **8.** 3 × 826

- **9.** 2 × 735
- **10.** 9 × 310
- 11.  $4 \times 724$
- **12.** 8 × 612
- 13-20. Name the products in examples 5 to 12 using the short method.

lame these products using the short method.

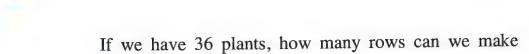
- **21.** 513 × 3
- **22.** 421 × 4
- **23.** 367
- **24.** 485 × 3
- **25.** 264
- **26.** 526

- olve the problems.
- 27. There are 341 seats in the movie theater. If all the seats were taken on 4 afternoons, how many people were at the movie?
- 28. If the ticket seller at the theater has 3 rolls of tickets, each with 450 tickets, how many is that in all?

 $\times$  6

# Using sets and numbers

with 9 plants in each row?



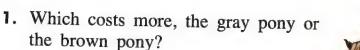
- Step 1: What are the sets? (A set of 36.)
- Step 2: What is happening to the set? (It is being separated to form sets of 9).
- Step 3: Write an equation using the set.  $36 \div 9 = \boxed{n}$  or  $\boxed{n} \times 9 = 36$ or  $36 \div \boxed{n} = 9$  or  $9 \times \boxed{n} = 36$
- Step 4: Solve the equation. ( $\overline{n}$  = 4 since  $9 \times 4 = 36$ ).
- Step 5: Fit the answer to the problem. (There will be 4 rows of 9 plants).

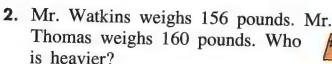
# EXERCISES

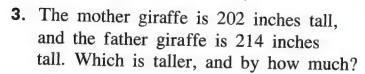
Use the 5-step method to solve these problems.

- 1. If one pencil costs 9¢, how many pencils can you buy for 72¢?
- 2. If a boy walked 4 miles in one hour, how many hours would it take him to walk 20 miles?
- **3.** Betsy had 50 stamps to fit on 8 pages of her stamp book. She could fit 6 stamps on a page. How many stamps did she have left over?
- **4.** If there were 4 children in each car, how many cars would it take to carry 32 children?
- 5. Eight books fit on each shelf. How many shelves will be filled and how many books will be left over if you put 75 books on the shelves?

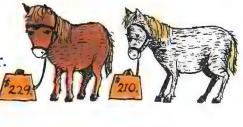
# Comparing sets

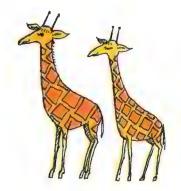


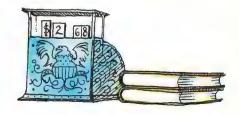


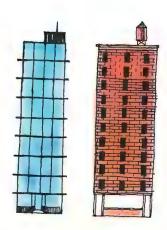


- **4.** Mary has 6 dimes and 7 cents. Sue has 4 dimes and 6 nickels. Which girl has more money?
- 5. Debbie has 1 dollar, 3 dimes and 4 cents. Does she have enough to pay for her books?
- **6.** Bill has 147 inches of rope. Will he have enough rope if he needs 12 feet of rope?
- 7. There are 9 stories in the glass building, and each story is 12 feet tall. The brick building has 11 stories, and each story is 10 feet tall. Which building is taller? How many feet taller?
- 8. Terry has 1 dollar, 5 dimes and 2 nickels to buy school supplies. Lawrence has 17 dimes. Who has more money?



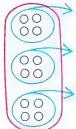






# Division

There are 3 sets of 4 in a set of 12. We write  $12 = 3 \times 4$  or  $12 \div 4 = 3$ .



We can take 3 sets of 4 from a set of 12 and get the empty set. We write  $12 - (3 \times 4) = 0$  or 12 - 12 = 0.

# EXERCISES

# Complete the charts.

	there are	in	equation
	2 sets of 6	1 set of 12	$2 \times 6 = 12$ or $12 \div 6 = 2$
1.	4 sets of 5	1 set of 20	$/4 \times \boxed{a} = 20 \text{ or } 20 \div 5 = \boxed{e}$
	7 sets of 8	1 set of 56	$7 \times \boxed{r} = 56 \text{ or } 56 \div 8 = \boxed{s}$
3.	9 sets of 10	1 set of 90	$9 \times \overline{\mathbf{u}} = 90 \text{ or } 90 \div 10 = \overline{\mathbf{v}}$
4.	6 sets of 12	1 set of 72	$6 \times x = 72 \text{ or } 72 \div 12 = z$

from	take	equation
1 set of 8	4 sets of 2	$8 - (4 \times 2) = 0$ or $8 - 8 = 0$
1 set of 15	5 sets of 3	$15 - (5 \times \boxed{e}) = 0 \text{ or } 15 - \boxed{n} = 0$
1 set of 32	8 sets of 4	$32 - (8 \times \mathbf{a}) = 0 \text{ or } 32 - 0 = 0$
	4 sets of 11	$44 - (4 \times \mathbf{r}) = 0 \text{ or } 44 - \mathbf{v} = 0$
	7 sets of 13	$91 - (7 \times \mathbf{S}) = 0 \text{ or } 91 - \mathbf{u} = 0$
	1 set of 8	1 set of 8

# Complete the sentences.

9. 
$$72 - (9 \times 8) = 72 - \mathbf{a} = \mathbf{e}$$

**10.** 
$$64 - (8 \times 8) = 64 - \mathbf{n} = \mathbf{0}$$

11. 
$$56 - (7 \times 8) = 56 - \overline{u} = \overline{v}$$

**12.** 
$$48 - (6 \times 8) = 48 - \boxed{z} = \boxed{a}$$

13. 
$$40 - (5 \times 8) = 40 - \overline{c} = \overline{u}$$

**14.** 
$$45 - (9 \times 5) = 45 - \vec{v} = \vec{x}$$
  
**15.**  $14 - (7 \times 2) = 14 - \vec{a} = \vec{r}$ 

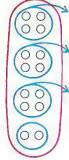
$$\overline{c}$$
 sets of 8 are in 72.

$$\overline{r}$$
 sets of 8 are in 64.



There are 3 sets of 4 in 1 set of 14 with 2 left over.

We write  $(3 \times 4) + 2 = 14$ .



If we take 3 sets of 4 from 1 set of 14, we get a remainder of 2.

We write  $14 - (3 \times 4) = 2$ .

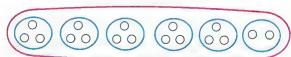
The 2 is called the remainder.

#### EXERCISES

Use the sets to complete these sentences.



in a set of 15 with c left over. The 3 left over is called the \_\_?\_.

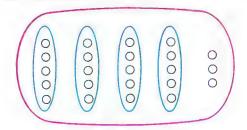


2. There are n sets of 3 in a set of 17 with r left over. The 2 left over is called the \_\_?\_\_.

# Complete these charts.

	in	there are	left over	equation
	1 set of 11	5 sets of 2	1	$11 = (5 \times 2) + 1$
3.	1 set of 18	2 sets of 8	a	$18 = (\mathbf{C} \times 8) + \mathbf{e}$
4.	1 set of 57	6 sets of 9	n	$57 = (0 \times 9) + \mathbf{r}$
<b>5</b> .	1 set of 47	7 sets of 6	S	$47 = (\mathbf{u} \times 6) + \mathbf{v}$
6.	1 set of 23	3 sets of 7	r	$23 = (\mathbf{x} \times 7) + \mathbf{z}$

	from	take away	remainder	equation
	1 set of 14	2 sets of 6	2	$14 - (2 \times 6) = 2$
7.	1 set of 44	5 sets of 8	a	$44 - (5 \times \mathbf{x}) = 4$
8.	1 set of 100	9 sets of 11	Z	$100 - (9 \times \mathbf{e}) = 1$
9.	1 set of 59	8 sets of 7	C	$59 - (8 \times \mathbf{r}) = 3$
10.	1 set of 25	6 sets of 4	r	$25 - (6 \times \mathbb{Z}) = 1$



There are 4 sets of 5 in a set of 23 with a remainder of 3. We write  $(4 \times 5) + 3 = 20 + 3$ = 23

To rename  $23 \div 4$  we can use this method.

5
4)23 Step 1: 
$$4 \times 5 = 20$$
20
3 Step 2:  $23 - 20 = 3$ 

The 5 is the quotient and the 3 is the remainder. We can check our work like this:

$$(4 \times 5) + 3 = 20 + 3$$
  
= 23

# EXERCISES

Name the quotients to complete these divisions. The first one is done for you.

1. 
$$9)\overline{29}$$
 $27$ 
 $2$ 

2. 
$$5)\overline{28}$$
 3.  $7)\overline{25}$   $25$   $3$   $4$ 

3. 
$$7)25$$

$$21$$

Name the remainders to complete these divisions. The first one is done for you.

6. 
$$4)\overline{25}$$
 $24$ 
 $1$ 
 $\overline{a}$ 
 $\overline{a}$ 
 $5$ 
 $8. 6)\overline{32}$ 
 $\overline{a}$ 
 $\overline{a}$ 

Name the quotients and remainders to complete these divisions. The first one is done for you.

$$\begin{array}{c}
 7 \\
 \hline
 11. \ 3)23 \\
 \hline
 21 \\
 \hline
 2
\end{array}$$

12. 
$$5)\overline{24}$$
 13.  $4)\overline{31}$  14.  $9)\overline{40}$  28 36

$$\begin{array}{c}
5)24 \\
20 \\
\hline
 \boxed{n}
\end{array}$$

Lynne does her division like this.



She checks her work like this.

### EXERCISES

Use Lynne's method to complete these divisions. The first one is done for you.

1. 
$$4)\overline{26}$$
 $\frac{24}{2}$ 

$$\frac{s}{z}$$

$$\frac{z}{o}$$

Use Lynne's method to name the quotients and the remainders.

**12.** 
$$6\overline{)38}$$
 **13.**  $8\overline{)50}$  **14.**  $9\overline{)65}$ 

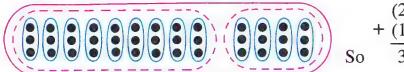
**16.** 
$$7)\overline{59}$$
 **17.**  $4)\overline{35}$  **18.**  $6)\overline{45}$  **19.**  $3)\overline{26}$ 

21-40. Use Lynne's check method in exercises 1-20.

Solve the problems.

- 41. Jane and her 6 friends have collected 60¢. If each of the 7 children is to get 8¢, how much will be left over?
- There are 19 apples for 6 children. How many apples will there be for each child and how many left over?

#### Division and the common factor rule

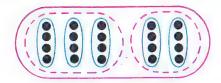


To find the number of sets of 3 in a set of 39 Carol wrote  $39 \div 3 = \boxed{n}$ . But she could not solve her equation. The greatest number she knew with 3 as a factor was 27. She knew that 27 + 12 = 39 and both 27 and 12 have 3 as a **common factor**. So she wrote three equations as we have done next to the sets above. Carol might have written equations like these to show how to use the **common factor**  $39 \div 3 = (27 \div 3) + (12 \div 3) = 9 + 4 = 13$ 

#### EXERCISES

Complete the equations for these sets.

1.



$$(12 \div 4) = \boxed{a}$$

$$+ (8 \div 4) = \boxed{c}$$

So 
$$20 \div 4 = \boxed{e}$$

$$(20 \div 5) = \boxed{a}$$

$$+ (10 \div 5) = \boxed{c}$$

$$\text{So } 30 \div 5 = \boxed{e}$$

Solve these equations.

3. 
$$(21 \div 3) = \boxed{a}$$
  
  $+ (12 \div 3) = \boxed{c}$   
  $33 \div 3 = \boxed{e}$ 

4. 
$$(49 \div 7) = n$$
  
  $+ (35 \div 7) = 0$   
  $84 \div 7 = r$ 

5. 
$$(12 \div 6) = \boxed{s}$$
  
  $+ (48 \div 6) = \boxed{u}$   
  $60 \div 6 = \boxed{v}$ 

Write the numerals to solve these equations.

**6.** 
$$45 \div 3 = (30 \div 3) + (15 \div 3) = \mathbf{a} + \mathbf{c} = \mathbf{e}$$

7. 
$$54 \div 3 = (30 \div 3) + (24 \div 3) = \mathbf{n} + \mathbf{0} = \mathbf{r}$$

**8.** 
$$64 \div 4 = (40 \div 4) + (24 \div 4) = \boxed{s} + \boxed{u} = \boxed{v}$$

**9.** 
$$72 \div 6 = (60 \div 6) + (12 \div 6) = \overline{v} + \overline{x} = \overline{z}$$

330 (three hundred thirty)

# Division and the common factor rule



Study these equations and number patterns.

$$(15 \div 3) = 5 + (12 \div 3) = 4 27 \div 3 = 9$$

$$\times 3 
5 15 
4 12 
9 27$$

# EXERCISES

Write the missing numerals.

1. 
$$(18 \div 3) = \boxed{a}$$
 $+ (9 \div 3) = \boxed{c}$ 
 $27 \div 3 = \boxed{e}$ 
 $\times 3$ 
 $n = 18$ 
 $0 = 9$ 
 $9 = 27$ 

3. 
$$(24 \div 4) = \boxed{r}$$
 $+ (12 \div 4) = \boxed{a}$ 
 $36 \div 4 = \boxed{n}$ 
 $\times$ 
 $4$ 
 $2$ 
 $24$ 
 $u$ 
 $12$ 
 $e$ 
 $36$ 

2. 
$$(35 \div 5) = \boxed{r}$$
  
 $+ (15 \div 5) = \boxed{s}$   
 $50 \div 5 = \boxed{u}$ 
 $\times \boxed{5}$   
 $\times \boxed{5$ 

4. 
$$(48 \div 8) = \boxed{c}$$
  
 $+ (24 \div 8) = \boxed{u}$   
 $72 \div 8 = \boxed{z}$ 
 $\times$ 

8

a 48

r 24

e 72

Complete these number patterns.

5.	×	4	
	a	28	
	r	16	
	z	44	

Write the missing numerals.

13. 
$$(40 \div 2) = \boxed{e}$$
  
  $+ (8 \div 2) = \boxed{a}$   
  $48 \div 2 = \boxed{r}$   $2$   $48$ 



To find the number of sets of 3 in a set of 42, Gordon set his work up like this:

Step 1: He thought: 42 > 30, and 30 is the largest product I know with 3 as a factor.

Step 2: He thought:  $3 \times 10 = 30$ .

**Step 3:** He thought: 42 - 30 = 12. **Step 4:** He thought:  $12 \div 3 = 4$ .

Step 5: So  $42 \div 3 = 10 + 4 = 14$ .

# EXERCISES

Copy and complete these divisions using Gordon's method.

Gordon knew that 7)84 is the working form for  $84 \div 7 = \boxed{a}$ . So 7 is the **given** factor, 84 is the product and 12 is the **missing factor**.

7)84	
63	9
21	
21	3
	12

# EXERCISES

Copy and complete these divisions using Gordon's method on page 332.

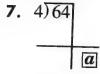
1. 4) 76 40 a 36 36 r 2. 7) 84 63 \$ 21 21 \$ 0 \$ 7

3. 8) 104 80 **u a z** 0 **x** 

4. 3) 57 a 10 27 e 9 0 c

5. 4)64 0 10 24 e 6 0 n 6. 2)38 0 10 18 r 9 0 s

Name the missing factors.



**8.** 3)54

**9**. 6)72

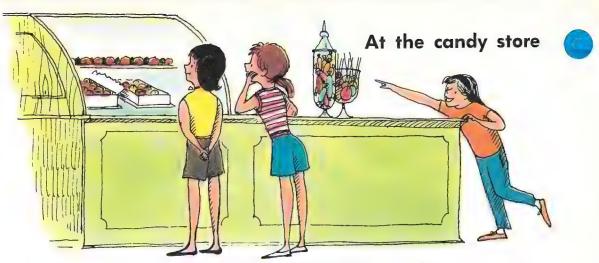
10. 9)108 r

11. 8)128

**12.** 7) 126

# Playing marbles

- 1. Eric has 26 marbles. If he puts the same number of marbles into 3 bags, how many will be left over?
- 2. If Eric had put his marbles into 5 bags with the same number of marbles in each bag, how many marbles would be left over?
- 3. Curt has 59 marbles. He wants to put the same number of marbles into 6 bags. How many will go in each bag, and how many will be left over?
- **4.** George has 45 marbles. He wants to give each of his 7 friends the same number of marbles. How many should he give to each friend, and how many would be left over?
- **5.** Donald has 29 marbles. He wants to give the same number of marbles to 3 of his friends. How many should he give to each friend, and how many would be left over?
- 6. Pete has 50 marbles. He put the same number of marbles into 8 bags and told his brother he would give him the number left over. How many did he give his brother?
- 7. Luke has 65 marbles. He wants to line them up in 9 rows. If each row is to have the same number of marbles, a. how many should he put into each row, b. how many marbles will be left over?



- 1. In the candy store Elizabeth saw 42 lollipops in 3 jars. Each jar had the same number of lollipops in it. How many lollipops were in each jar?
- 2. Elizabeth also saw 64 candy bars in 4 boxes. If each box had the same number of candy bars, how many candy bars were in each box?
- **3.** Elizabeth bought 36 licorice sticks. The salesman put an equal number into each of 4 bags. How many are in each bag?
- **4.** Sybil counted 84 marshmallows in 7 boxes. If each box had the same number of marshmallows, how many were in each box?
- 5. Sybil bought 28 chocolate drops. The salesman put them into 2 bags. If each bag had the same number of chocolate drops, how many were in each bag?
- **6.** Laurie counted 135 chocolate covered cherries in 5 boxes. If each box contained the same number of cherries, how many were in each box?
- 7. Laurie bought 144 chocolate covered cherries. The salesman put an equal number into each of 8 bags. How many were in each bag?

# Division

Study these divisions done twice by the methods you have learned.

#### Method A

#### Method B

Is it easy to divide with 10 as a factor? Is it easy to name a sum such as 10 + 4?

# EXERCISES

Copy and complete these examples.

Use method B to name the missing factors.

7. 
$$54 \div 3 = a$$

8. 
$$57 \div 3 = [c]$$

9. 
$$108 \div 9 = e$$

10. 
$$60 \div 4 = [r]$$

11. 
$$36 \div 2 = n$$

12. 
$$95 \div 5 = 0$$

13. 
$$80 \div 5 = \boxed{s}$$

14. 
$$84 \div 6 = [u]$$

15. 
$$38 \div 2 = |v|$$

16. 
$$28 \div 2 = [x]$$

17. 
$$96 \div 8 = \boxed{z}$$

18. 
$$98 \div 7 = [a]$$

19. 
$$96 \div 6 = [r]$$

**20.** 
$$104 \div 8 = [\overline{u}]$$

**21.** 
$$32 \div 2 = n$$

**22.** 
$$68 \div 4 = \boxed{c}$$

23. 
$$75 \div 5 = u$$

**24.** 
$$120 \div 8 = \mathbf{r}$$



Study these methods for naming the missing factor.

0

Step 1:  $30 \div 3 = 10$  Write 10. Step 2: 42 - 30 = 12Step 3:  $12 \div 3 = 4$  Write 4. Step 4: 12 - 12 = 0Step 5: 10 + 4 = 14

 $42 \div 3 = 14$ 

C.  $3)\overline{42}$ 

Step 1:  $30 \div 3 = 10$  Write 1 in the tens' place.

**Step 2:** 42 - 30 = 12

Step 3:  $12 \div 3 = 4$  Write 4 in the ones' place.

**Step 4:** 12 - 12 = 0,  $42 \div 3 = 14$ 

# EXERCISES

Use method A to name the missing factors.

1. 4)72

**2.** 5) 165

**3**. 3)48

**4**. 6) 156

**5**. 5)110

**6**. 8)168

**7.** 4)132

**8.** 6) 132

**9.** 7)77

**10**. 4) 148

Use method B to name the missing factors.

11. 3)96

**12**. 4) 128

**13**. 5)80

**14.** 6) 132

**15**. 7)161

**16.** 7) 147

**17.** 9)189

**18**. 7)238

19. 8)248

**20**. 9)279

Use method C to name the missing factors. Use 10, 20, or 30 as your first factor.

**21.**  $36 \div 3 = a$ 

**22.**  $45 \div 3 = e$ 

23.  $48 \div 4 = \boxed{n}$ 

**24.**  $96 \div 6 = [r]$ 

**25.**  $120 \div 6 = \boxed{s}$ 

**26.**  $145 \div 5 = \overline{u}$ 

**27.**  $56 \div 4 = \overline{v}$ 

**28.**  $60 \div 5 = x$ 

**29.**  $138 \div 6 = \mathbf{z}$ 

Study this example for naming the quotient.

14 3)44

 $30 \div 3 = 10$  Write 1 in the tens' place. Step 1: 30

14

Step 2: 44 - 30 = 14

12

Write 4 in the ones' place. Step 3:

14 - 12 = 2Step 4:

When 44 is divided by 3, the quotient is  $\square$ , the remainder is 2.

$$44 = (14 \times 3) + 2$$

# EXERCISES

Complete these divisions. The first one is done for you.

13 1. 5)69 50

19 15

v**2.** 3)71

60 11

 $\propto$ 

z**3**. 8)173

160 13

8  $\boldsymbol{a}$ 

|c|4. 6) 135

120 15

12 |e|

 $\boldsymbol{n}$ **5.** 4) 151

120 31 28

0

e

r **6.** 9)194 180

> 14 S

**7.** 7)150 140 10

|u|

v

 $\boldsymbol{x}$ **8.** 4) 135

120 15 12

|z|

|a|**9**. 5)156

150 6

|c|

**10**. 7)220

210 10 7

|n|

Name the quotients and the remainders.

11. 3)35

**12.** 4)376

**13**. 6) 129

**14.** 8)295

**15**. 5)138

6)495 16.

**17.** 7)680

**18**. 5) 193

**19**. 9)395

**20**. 8) 139

4)259 21.

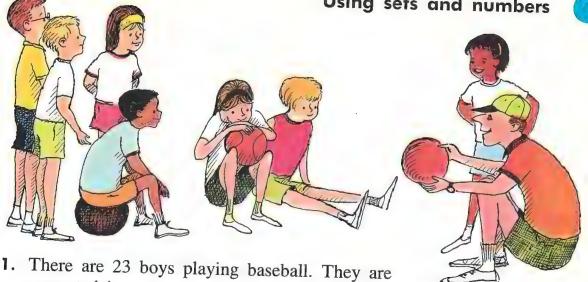
**22.** 3)613

**23.** 7)187

**24.** 9)243

**25**. 8) 566

# Using sets and numbers



separated into two teams with 9 boys on each team. How many boys are left over?

- 2. There are 31 girls to separate into 4 teams with 7 girls on each team. How many girls are left over?
- 3. If there are 45 girls and boys all together, a. how many teams with 8 children can be formed? b. how many children will be left over?
- 4. There are 75 boys and girls all together. The teacher wants to make 6 rows of children for a relay race.
  - a. How many children will there be in each row?
  - b. How many children will be left over?
- i. There are 175 books to go into 8 classrooms. The same number of books is to go into each room.
  - a. How many books will each class have? b. How many books will be left over?
- . The teachers want to put the same number of children on 6 buses. If there are 186 children,
  - a. how many children should go on each bus?
  - b. are there any children left over?

# CHECKING UP

Complete these examples.

$$\begin{array}{c|c}
1. & 14 \\
+ & \boxed{a} \\
\hline
42
\end{array}$$

4. 
$$724$$
 $- u$ 
 $362$ 

6. 
$$\begin{array}{c} 5 \\ \times 3 \\ \hline 126 \end{array}$$

7. 
$$\frac{v}{\times 4}$$

8. 
$$\frac{n}{\times 4}$$

9. 
$$\frac{z}{\times 4}$$

10. 
$$a$$

$$\times 9$$

$$423$$

Solve these equations.

**21.** 
$$4 \times e = 72$$

**22.** 
$$68 \div a = 4$$

**23.** 
$$69 \div \mathbf{n} = 23$$

Copy and complete these divisions.

Solve these problems.

- 27. If you have 176 pencils and want to put them into packets, each containing 8 pencils, how many packets will you fill with pencils?
- 28. If there are 200 nails in 8 packets and each packet contains the same number of nails, how many are in each packet?
- 29. How many weeks are there in 186 days? How many days are left over?

# **EXTRA FOR EXPERTS**



- Multiply 37 by 3, by 6, by 9. What do you notice? Can you name a. the product 37 × 24 without multiplying? b. the missing factor 555 ÷ 37 without dividing?
- 2. Multiply the number 12,345,679 by any number less than 10, then multiply the product by 9. What do you notice?
- **3.** Multiply the number 142,857 by any number less than 7. What do you notice? What is the product when you multiply by 7?
- 4. Name the missing factors using this method.

$$91 = 70 + 21$$
, so  $7)91 = 7)70 + 21$ 

- a.  $3)\overline{54}$
- **b.**  $4)\overline{76}$
- c. 5)130
- d.  $4)\overline{68}$

- **e.**  $6)\overline{96}$
- f. 7)105
- **g.** 8)112
- **h.** 9)126

Just For Fun

5. Billy said, "If you write a 4-place numeral, I can write another numeral so that the sum of the two numbers we have named is 9999." Kate wrote 4762. Underneath this Billy wrote 5237. Is it true that 4762 + 5237 = 9999? Can you see how Billy did this trick?



6. Using 1, 2, 3, 4, 5 write names for the numbers 1 to 10. Each digit is used once and only once in each name. Here is one way to show 7, using 1, 2, 3, 4, 5: 4 + 5 + 2 - 3 - 1. Here is one way to show 9: 5 + 4 + 3 - 1 - 2.

Abacus (p. 9)

Rods and beads used to show place value.

Addend (p. 34)

In addition, the numbers used to name the sum.

Addition (p. 32)

The renaming of two addends as a sum.

Angle (p. 74)

Two rays with a common end point.

Area (p.88)

The measure of a region together with the unit used for measurement.

Braces { } (p. 2)

Symbol used to enclose the members of a set.

Cardinal number (p. 4)

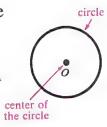
The number that tells how many objects are in a set.

Center (of a circle) (p. 84)

The point in the exact middle of a circular region. The distance from the center to every point on the circle is always the same.

Circle (p. 84)

A circle is a simple closed curve with all the points a fixed distance from a given point (the center).



Closed curve (p. 67)

A curve that begins and ends at the same point.

Common factor (p. 192)

A number which is a factor of two or more other numbers. 4 is a common factor of 8 and 12.

Common Factor Rule (p. 252)

To name the sum of two addends with a common factor, such as

$$(3\times5)+(3\times7)$$

we may multiply then add

$$(15 + 21 = 36)$$

or add then multiply

$$(5 + 7 = 12, 12 \times 3 = 36).$$

Common multiple (p. 288)

A number that is a multiple of two or more numbers.

Corner point (pp. 74, 80)

The point where two rays meet to form an angle. Also, the point where two sides of a plane figure meet.

# Counting numbers (p. 10)

The numbers in  $\{1, 2, 3, 4, ...\}$ .

# Lube (p. 293)

A space figure that looks like a box. It has six square regions with equal measures as sides.



#### Jurve (p. 62)

A set of connected points that form a path.

# ylinder (p. 292)

A space figure that looks like a can.

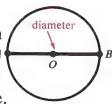


# iagonal (p. 83)

A line segment that joins two corner points of a plane figure but is not one of its sides.

# iameter (of a circle) (p. 85)

A diameter of a circle is a line segment that has both end points on the circle and A passes through the center of the circle.



# fference (p. 33)

The missing addend in a subtraction.

# Digit (p. 6)

A symbol used to write numerals. In our system of writing numerals, the digits are 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0.

# Division (p. 130)

A number operation that is the opposite of multiplication.

# Ellipse (p. 311)

A simple closed curve that is oval in shape.

# Empty set (p. 4)

The set that has no members. Its cardinal number is zero.

# Equation (p. 34)

A mathematical sentence stating that two numerals name the same number.

# Equivalent sets (p. 4)

Sets with the same cardinal number.

# Even number (p. 226)

A whole number that has 2 as a factor.

# Expanded numeral (p. 8)

A numeral such as in

$$30 + 6$$

or

$$7 + \frac{3}{4}$$
.

**Factor** (p. 126)

A number that is multiplied by another number to give a product. In the equation  $2 \times 3 = 6$ , 2 and 3 are factors.

Finite set (p. 222)

A set whose members can be listed and counted.

Fraction (p. 160)

A name for a fractional number.

Fractional number (p. 160)

A number that tells how a part compares with a whole; the whole may be a region, a single object, or a set of objects.

Graph (p. 302)

A picture used to show data. Different types of graphs are bar, line, and circle.

Grouping Rule for Addition (p. 40)

The way in which the addends are grouped does not affect the sum.

$$(5+4)+3=5+(4+3).$$

Grouping Rule for Multiplication (p. 255)

The way in which the factors are grouped does not affect the product.

$$(2 \times 3) \times 4 = 2 \times (3 \times 4).$$

Infinite set (p. 223)

A non-empty set which is not finite. All of its members cannot be listed and counted.

Lattice (p. 306)

Number lines meeting at right angles to each other, used with number pairs to locate points on a plane.

Line (p. 62)

A straight curve that goes on without end in either direction.

 $\bar{B}$ 

Line segment (p. 62)

Part of a line  $\frac{1}{A}$  with 2 endpoints.

Matching angles (p. 288)

Two or more angles having the same size.

Measure (p. 68)

A number that tells how many units match an object for the property being measured: length, area, or volume.

Multiple (p. 224)

The multiple of a number is a product of that number and a whole number. The multiples of 3 may be found by multiplying 3 by the members of the set  $\{0, 1, 2, 3 ...\}$ .

# Multiplication (p. 130)

The renaming of two or more factors as a product.

# Number (p. 4)

An idea of "how many?"

#### Number line (p. 10)

A line with points labeled by numbers.

### Number pair (p. 186)

Two numbers such that the first number and a rule give the second number. Number pairs are also used to locate points on a lattice or number plane (p. 298).

#### Numeral (p. 6)

A name for a number.

### Odd number (p. 226)

A whole number that does not have 2 as a factor.

#### One-to-one matching (p. 4)

Pairing the members of two sets so that every member of the first set is matched with a member of the second set, and no members in either set are left over.

# Order rule (p. 313)

The order in which you add addends or multiply factors does not change the sum or the product.

# Ordinal number (p. 12)

A number such as *first*, *second*, and so on, tells "which one in order."

# Parallel lines (p. 76)

Two or more lines in the same plane that never meet. The distance between them is always the same.

# Parallelogram (p. 291)

A four-sided plane figure with both pairs of opposite sides parallel.

# Pairing (p. 204)

Matching a member of one set with a member of another set.

# Plane (p. 66)

A flat surface that goes on without end.

# Point (p. 63)

A location in space, often shown by a dot.

# Prime number (p. 232)

A whole number greater than 1 that has only two factors, itself and 1.

# Product (p. 127)

When two numbers are multiplied together, the result is a product, as 81 is the product in  $9 \times 9 = 81$ .

# Quadrilateral (p. 80)

A four-sided plane figure.

# Quotient (p. 338)

Another name for the missing factor in division.

# Radius (of a circle) (p. 85)

A radius of a circle is a line segment with one end point on the circle and the other end point on the center of the circle.

#### Ray (p. 64)

Part of a line that has one end point and goes on without end in one direction.



#### Rectangle (p. 82)

A quadrilateral with 4 right angles.

#### Region (p. 87)

The plane area inside a simple closed curve.

# Remainder (p. 338)

The number remaining in a division when the greatest multiple of the divisor has been subtracted from the dividend. 5 is the remainder in  $\frac{4}{6 \cdot 29}$ .

# Right angle (p. 75)

An angle that looks like a square corner.



# **Set** (p. 2)

radius

A collection of objects or ideas.

# Set of factors (p. 230)

The group of all the whole numbers that can be factors of a given number.

# Simple closed curve (p. 67)

A closed curve in a plane such that if you draw a picture of the curve, your pencil will return to the starting point, it will never leave the paper, and it will not go through any point twice.

# Sphere (p. 292)

A space figure that looks like a ball.

# **Square** (p. 82)

A rectangle with 4 sides the same length.



# Subset (p. 3)

A is a subset of B if the members of A are all members of B.

# Subtraction (p. 32)

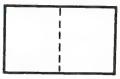
The number operation that is the opposite of addition.

# Sum(p. 34)

The number that is the result of renaming addends.

# Symmetrical shape (p. 286)

A shape that can be folded in half so that both halves match exactly.



Triangle (p. 80)

A three-sided plane figure.



Volume (p. 238)

The number of space units that fit inside a space figure.

Whole number (p. 52)

A number in  $\{0, 1, 2, 3, \ldots\}$ .

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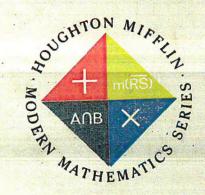
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